Another Look at the Single-Tuned Circuit

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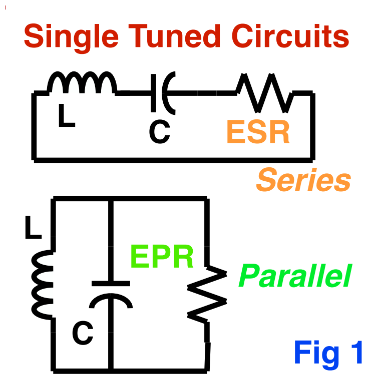
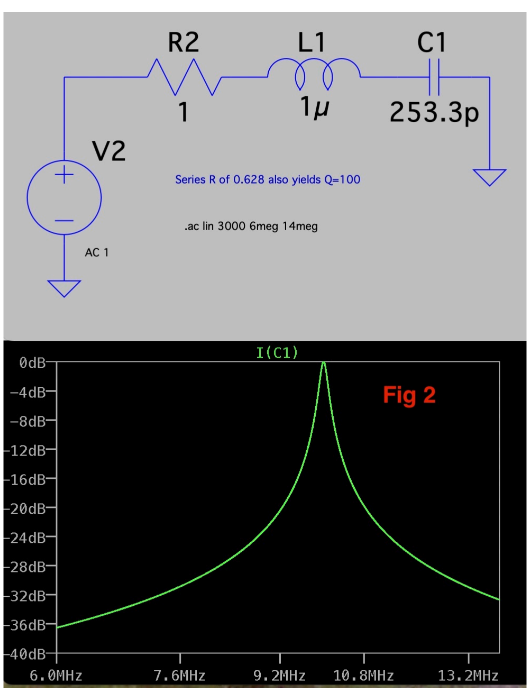
I've been interested in electric networks and especially filters for a long time. I've emphasized the complex multi resonator circuits that offer the performance we want for our communications equipment. This may have been an oversight. The simpler single tuned circuits illustrate many characteristics that are central to all filters and can even be practical for some applications.



This note will take a look at this most basic filter, the single tuned circuit. Both LC and crystal filters will be considered. We'll use the mathematics that are actually at the core of the subject. Figure 1 shows two forms for the RLC circuit using series and then parallel resonance. The two forms differ in the way resistance appears in the circuit. The resistance is often specified as ESR or EPR, indicating Equivalent Series Resistance or Equivalent Parallel Resistance.

The Series Tuned Circuit

An example RLC is shown in Fig 2. This series tuned circuit is resonant at 10 MHz.



The plot of this series tuned circuit shows current Vs frequency. This circuit has a peak exactly at 10 MHz. The response is 3 dB below the peak at 9.922 and 10.079 MHz. The difference of 0.157 MHz is termed the circuit bandwidth, B.

At this point we introduce the critical parameter Q=f_c/B where f_c is the center frequency and B is the bandwidth. Both B and f_c have the same units. See p78 of Skilling[1]. It is common to think of Q as a figure of merit for an inductor, capacitor, or resonator. It is more than this. It's a numeric that we use in the design of tuned circuits as well as components. We will usually use Q with a subscript for specific applications.

$$Q = \frac{f_c}{B} \qquad \text{Eq1}$$

[1] H Skilling, **Electric Networks**, Wiley, 1974.

Q was defined by a frequency and bandwidth. It is also related to the ratio of a reactance to a related resistance. ω is termed the *angular frequency* and is related to frequency f by $\omega=2\pi f$. Subscripts will be applied when useful. Q is often applied to individual components. So $Q_L = \omega L/ESR = X_L/ESR$ would describe an inductor where ESR is the equivalent series resistance of that part. X_L is the inductive reactance. ESR is NOT (usually) the series R that would be measured with a DC ohm meter. Rather, it is the resistive component of the impedance measured near or at the frequency of an application. The Q_C of a capacitor is similarly defined when capacitive reactance is substituted for X_L.

$$Q = \frac{\omega L}{R} \qquad Eq2$$

A series tuned circuit becomes a single resonator filter with the circuit of Fig 3. The capacitor value is just found with the familiar formula.

$$C_o = \frac{1}{\omega^2 L} \qquad Eq3$$

Consider an example, an LC filter centered at 7.1 MHz with a bandwidth of 100 kHz.

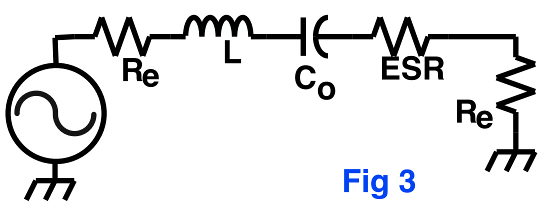


Fig 3

This will be based on an inductor with value 10 μH with Q_U=200 where the u subscript emphasizes that this describes unloaded Q. The resonator capacitor is then 50.25 pF. The total resistance in the circuit is $2R_e$+ESR. The inductive reactance divided by the total resistance will determine a Filter Q. But this filter Q is already determined by the frequency ratio, Q_f=7.1/0.1 = 71. This produces the design equation where the "e" subscript means "end" resistance. All equations in this note use fundamental units.

$$R_e = \frac{\omega L}{2}\left(\frac{1}{Q_f} - \frac{1}{Q_u}\right) \qquad Eq4$$

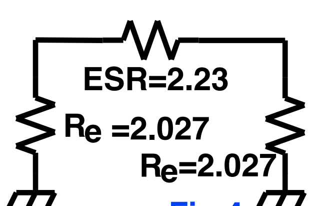


Fig 4

An abbreviated "at resonance" version of the design is in Fig 4 where the L and C are eliminated and the resistor values are shown. The source generator is eliminated to emphasize the loading function of the source and load resistors in the more familiar form of Fig 3.

The value for $ESR = \omega L / Q_U$ is what we might expect from our intuition, which leads to filter loss. The end resistance values are often not practical. A 50 ohm termination would be more comfortable. A 50 ohm or similar resistor can be forced to look like a much lower resistance if it is paralleled by a suitable capacitor, shown in Fig 5. The equations for the end capacitor C_e and that of the related C_{es} are also presented. The significance of C_{es} is illustrated in Fig 6, leading to a tune capacitor, C_t.

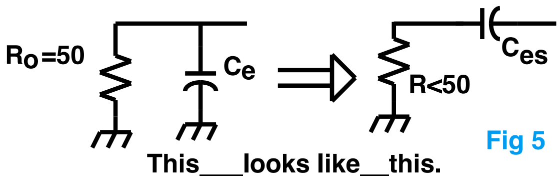


Fig 5

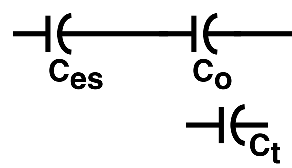


Fig 6

$$C_e = \sqrt{\frac{R_0 - R_e}{R_e \omega^2 R_0^2}} \qquad \text{Eq5}$$

$$C_{es} = \frac{(C_e \omega R_0)^2 + 1}{C_e \omega^2 R_0^2} \qquad \text{Eq6}$$

$$C_T = \frac{C_o C_{es}}{C_{es} - 2C_o} \qquad \text{Eq7}$$

The data for the final filter : C_e = 2181 pF, C_{es} = 2274 pF, C_o = 50.25 pF, L = 10 μH, C_t = 52.57 pF. The final series tuned filter is shown in Fig 7 with a calculated response in Fig 8.

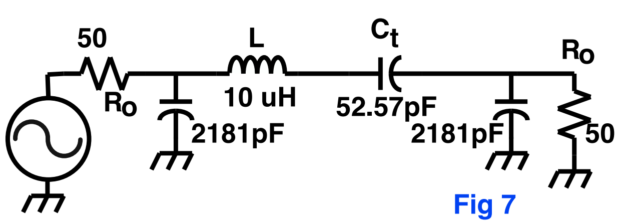
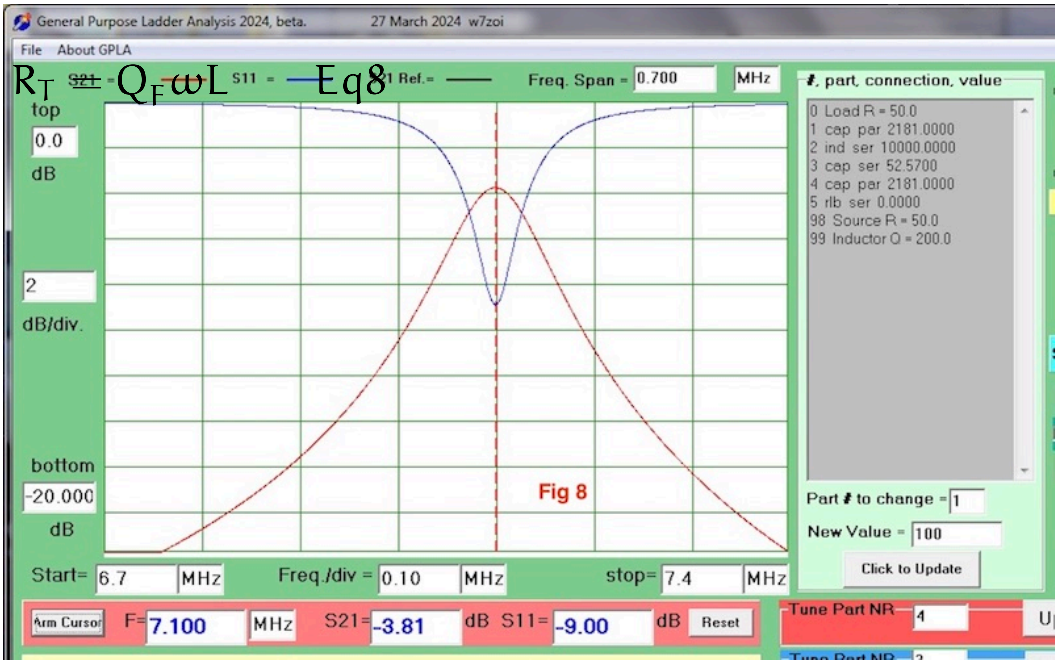


Fig 7

The equations in this article were derived in another note on our web site.[2]

[2] https://w7zoi.net/filters/genfil.pdf See section 3.

$$R_T = Q_F \omega L \qquad \text{Eq8}$$



Alas, there is a problem. This topology with a series tuned circuit does not always work. Assume a similar 7 MHz filter, but with a 20 µH inductor and a bandwidth of 1 MHz. The termination resistance is now greater than 50 ohms leading to imaginary or even infinite calculation results. This series tuned circuit filter can no longer be built, suggesting the need for a different topology.

The Parallel Tuned Circuit

When we cannot realize a series tuned circuit, we use a parallel resonator, shown in Fig 9. A parallel resistor R_u models the loss related to an unloaded Q_u.

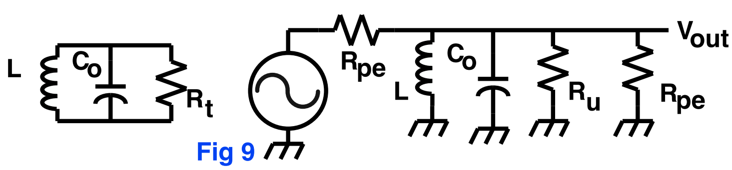


Fig 9

The filter bandwidth sets the net resistance R_t. We again wish to have equal loading at each end which determines the parallel end resistor R_{pe}. We include p in the subscript of R_{pe} to indicate a parallel resistance. The following equations are used:

$$R_U = Q_U \omega L \qquad \text{Eq9}$$

$$R_{pe} = \frac{2 R_U R_T}{(R_U - R_T)} \qquad \text{Eq 10}$$

Recall in the series tuned circuit presented earlier we made a moderate resistance R_o look like a much smaller resistance by paralleling it with a capacitor. The matching transformation now used with the parallel tuned circuit uses a series capacitor to make R_o look like a higher resistance, shown in Fig 10. Equation 10 relates R_o to R_{pe}. The required series C_{pc} is determined by Eq 11. Whenever we use a capacitor to transform R_o (usually our familiar 50 ohms) to a lower or higher value, the added capacitor detunes the resonator. This was shown in Eq 6 for the series tuned circuit. In the parallel TC case, the series capacitor means that the net tuning C_o for the resonator must be reduced by C_{pt}, Equation 12, yield a net C_{tune} shown in equation 13.

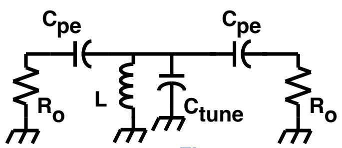


Fig 10

$$C_{pe} = \frac{1}{\omega\sqrt{R_o R_{pe} - {R_o}^2}} \qquad \text{Eq 11}$$

$$C_{pt} = \frac{C_{pe}}{(R_o \omega C_{pe})^2 + 1} \qquad \text{Eq 12}$$

$$C_{tune} = C_0 - 2C_{pt} \qquad \text{Eq 13}$$

Consider an example. We use the same inductor as before, 10 μH with Q_u=200, 50 ohm terminations, center frequency of 7.1 MHz with a 100 kHz bandwidth for Q_F=71. Intermediate output parameters are R_u=89.2K, R_T=31674, R_{pe}=98.2K, C_0=50.249pF. The final circuit is in Fig 11. K=1000, all equations use fundamental units.

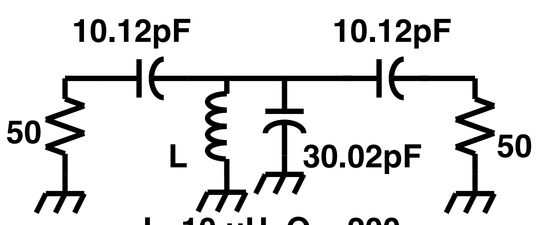
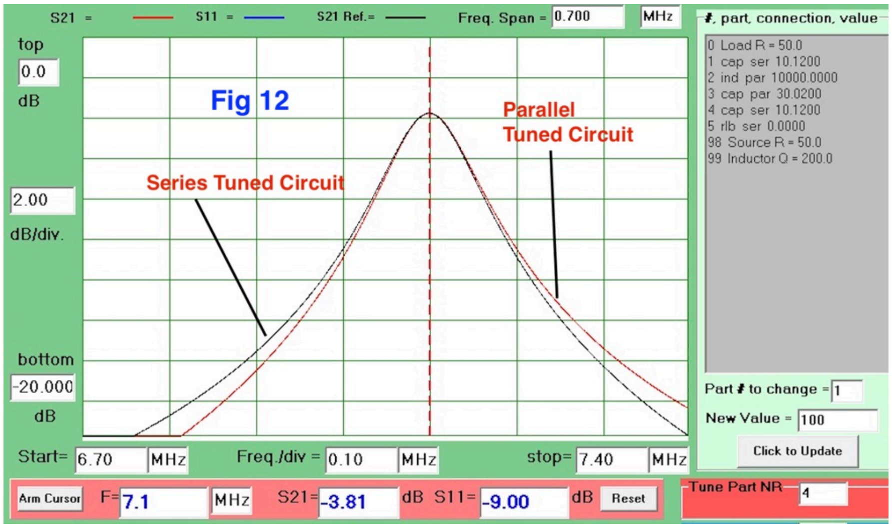


Fig 11

Fig 12 shows the response for this filter in red. Also included is the response for the earlier series tuned circuit, shown in Figures 7 and 8. There is a slight difference in the filter shapes, but it is small. The same inductor, 10 μH with Q_u=200, is used in both the series and the parallel tuned circuits, so the insertion loss is the same at 3.81 dB. There is nothing special about the inductance value. It was picked because it was a part pulled from the junk box.



A Single Crystal Filter

A quartz crystal has an equivalent circuit that is nearly the same as the LCR circuit of Fig 3. The crystal model also contains a small parallel capacitance that is across the the LCR. This small capacitance, usually just a few picofarads, can be ignored in many designs, for it has little impact for narrow bandwidth applications. The exact design equations that were used to design a single series tuned circuit can now be used for a simple crystal filter. We illustrate this with a 9 MHz filter. A batch of crystals was on hand for an earlier project[3] where we had characterized the parts.

The 9 MHz crystal had a motional inductance if 0.01576 H with an unloaded Q_U=124,000. These were average parameters for a batch of 35 crystals. Consider a single crystal filter using one of these parts. Assume that the filter will have a 3 dB bandwidth of 200 Hz. Using prior equations, the filter Q will be

$$Q_F = \frac{9 * 10^6}{200} = 45000$$

Using earlier Q measurements, we calculate ESR=7.19. The parallel capacitance was 4.9 pF. The motional capacitance, which will become the series C in the single tuned filter, is calculated from the 9 MHz resonance as 19.84 * 10^{-15} Farad. Parallel C and ESR are not used in the design, but will be used to evaluate the resulting filter. The basic filter is shown in Fig 13.

Equation 4 is used to calculate the end terminating resistance for this filter, resulting in R_e=6.31 ohms. We want to put this filter in a 50 ohm environment, Eq 5 is used to calculate C_e=930 pF. The final crystal filter schematic is shown in Fig 14 with a response plot in Fig 15. There are two plots in Fig 15. That in red uses 930 pF shunt

[3] See. https://w7zoi.net/filters/9megxfils.pdf The crystal is Mouser part number 449-LFXTAL064523BULK

capacitors while the black plot uses 1000 pF standard values, a practical value when building such a filter. The insertion loss changes a little when making this change.

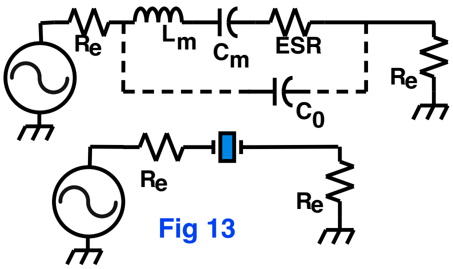


Fig 13

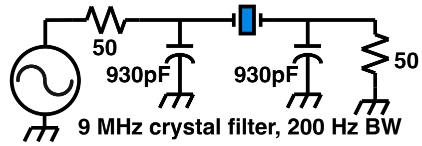
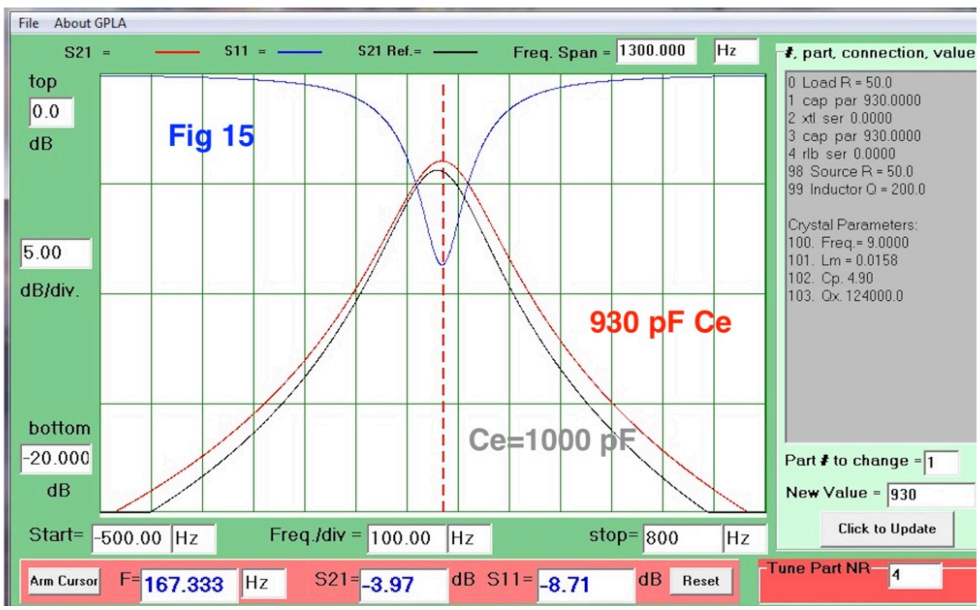


Fig 14



Concluding Thoughts

This has mainly been a design case-study. The goal was not as much to produce practical designs as it was to review the single tuned circuit as a part of the greater chore of designing and building higher order narrow bandwidth filters. That said, these simple filters can still be useful and easy to built. For example, the crystal filter design presented might be useful as a noise suppression filter to follow an IF amplifier in an experimental receiver.

The circuits presented have assumed that the only loss is related to the inductor part of the resonator. That's often an incomplete assumption, especially when using SMT capacitors. Even some leaded parts that we formerly assumed to be nearly ideal are compromised.

The filters presented have used equal terminations at the two ports, but this is not necessary. One may well use different terminations as part of an impedance transforming circuit. Indeed, an interesting variation is a single tuned circuit that looks like a series resonator form at one end, but a parallel circuit when viewed from the opposite port. This topology was the subject of a note in an earlier effort.[4]

[4] See QEX, June, 1993.