Simple Quartz Crystal Models: A Review

Wes Hayward, w7zoi, 2 May 2017

A recent Internet posting ask about quartz crystals and the way the properties, mainly stability, change as the package and size change, assuming the variations have the same frequency. It is an interesting question, one that is not always understood. It's been discussed before, but those discussions are lost. We thought it would be interesting to review the problem again.

So, what does a crystal look like? We know about the external packages, but it's what's inside that counts. The crystal is essentially a disc or slab of quartz. Quartz is a naturally occurring crystalline mineral of Silicon and Oxygen, SiO_2. Most of the "crystals" we use for communications start as synthetically grown structures. The traditional shape for our communications part is round, although this is changing. Indeed, it is part of the situation we consider here. The traditional disc may be half an inch in diameter, although it is usually smaller, perhaps a quarter of an inch. The thickness is typically in the 5 to 20 mil region. The quartz disc is coated with a metal. Attached wires allow a voltage to be applied that will create an electric field within the quartz material. A cross-section figure is shown below.

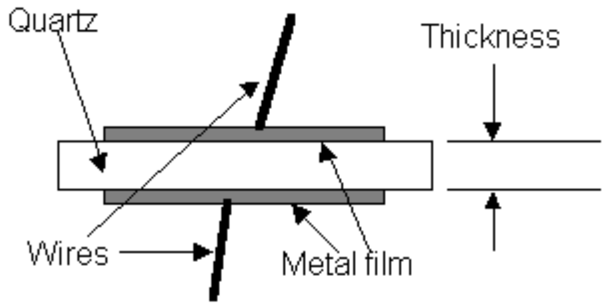

Fig 1. Cross section view of a quartz crystal.

We start our discussion with the simple physics of the piezoelectric effect. It may be very difficult to physically see this, but if we hook a battery between the electrodes, there will be a motion. The applied voltage causes an electric field to go between the plates, but the motion or displacement of quartz material is at right angles to this electric field. The behavior is otherwise linear. That is, the higher the applied voltage, the greater the displacement.

Many dielectric materials display the piezoelectric effect. But Quartz is especially stable with temperature, so it is used in frequency sensitive applications.

The piezoelectric effect finds application with DC (e.g., strain gauges), but our present interest is AC behavior. Like the DC, an applied AC voltage will also generate a displacement or motion. The magnitude of this motion changes with the frequency of the applied signal.. If we sweep the frequency of the applied voltage, we will find that the mechanical motion and the current from our generator will be a maximum at one resonant frequency. The Q of this resonance can be extremely high. Measuring the current (including phase) and comparing it with the driving voltage allows us to model the crystal. It looks like a series tuned circuit. That is, a series tuned circuit with the same voltage will produce exactly the same current.

The mechanical structure we have described in Fig 1 has a second, less complicated electrical characteristic. It is a simple parallel plate capacitor. The piezoelectric effect relates to crystal properties and is the basis for the observed resonance. In contrast, any dielectric that is plated with metal will become a capacitor.

We can combine the two effects described to form a more complete electrical model, shown below in Fig 2. The high Q series resonance is the interesting property. It's modified by the parallel C0. The series resonance has a finite Q. The finite Q means loss, which is usually modeled as a resistance. We can represent the crystal loss with a resistance in series with the series L and C.

We term the series resonant L and C to be "motional" elements, for they are related to the motion of the quartz with applied electric field. We'll call these Lm and Cm. The parallel capacitance is called C0. The motional resistance is called Rm, or alternatively, ESR for *equivalent series resistance*.

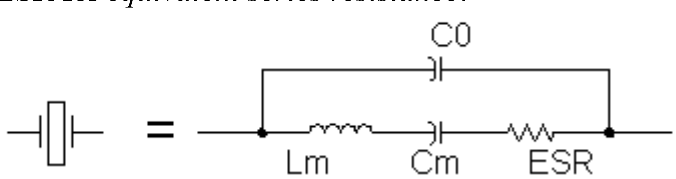

Fig 2. Electrical Model of a Quartz crystal. ESR is "Equivalent Series Resistance," also called Rm for "motional resistance." C0 is the capacitance associated with the simple parallel plate characteristic and can be measured at low frequencies, far below the resonance of the motional components, Lm and Cm.

What are some of the L and C values? They are very unusual if we think in terms of descrete components. For example, I have some 5 MHz crystals in my junk box. That is, the series resonance of Lm and Cm is at 5 MHz. The inductance has a value of Lm=0.1 Henry. We usually think of an inductor to be used at 5 MHz as having values measured in microhenrys, so the crystal L is higher by a factor of 100,000. Resonance calculations tell us that the motional capacitance tuning this monster inductance to 5 MHz is a very tiny .01 pF. More exactly, it's 10.132 fF where the f in fF stands for femto, which stands for 10^{-15}. The parallel C is C0=2.2 pF.

The Q of my 5 MHz crystal was measured as about 200,000. If we use the usual formula that relates series R to Q for a series tuned circuit, we find that the series resistance is Rm=15.7 Ohms. Our crystal model is now complete: A 0.1 Henry inductor, a capacitor of .010132 pF, and a 15.7 Ohm equivalent series resistor, all placed in series. The series network is paralleled by the C0.

A parallel resonance also appears in measurements. This resonance disappears when C0 is eliminated from the model. Fig 3 shows the overall response.

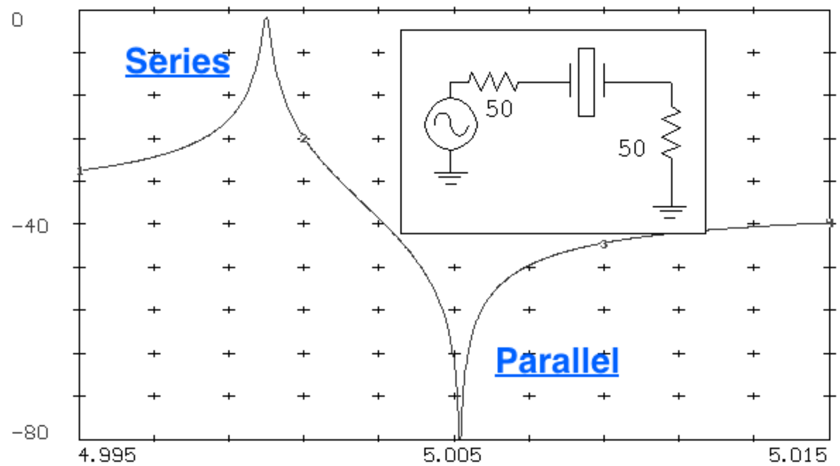

Fig 3. Calculated response of a 5 MHz crystal showing both series and parallel resonance.

The values presented above are the numbers for the raw quartz slab with metal plating. The quartz is quite thin, about 13 mils for this crystal. When we put this blank in a metal can, the C goes up slightly, but there is not much change beyond this. The parallel C we measure for our packaged part is up to around 3 pF. The package was type HC-49.

There are a couple of more details that will complete the modeling process. These further tie the physical characteristics to the electrical properties. First there is a firm relationship between the motional capacitance and the parallel capacitance. Specifically, C0=220 Cm. This is not a casual observation, but is a detail that emerges from the underlying theory that describes the quartz crystal[1].

The metal plates on a crystal may not cover the entire area. In fact, it is most common to have metallization only near the middle of the quartz disc. Controlling the fraction of the quartz that is covered by metal allows the motional parameters to be controlled.

See V. E. Bottom, **Introduction to Quartz Crystal Unit Design**, Van Nostrand Reinhold, 1982. A set of equations on page 98 describes all of the motional [1]parameters. Calculation of C0/Cm reduces to an expression that has no geometric details, but only more fundamental physical constants.

The final detail is perhaps the most important: The resonant frequency of our crystal is inversely proportional to the thickness of the quartz, but very little else. If we cut the quartz thickness in half, the resonant frequency will go up to 10 MHz. Doubling the thickness moves the frequency to 2.5 MHz[2].

What does this mean for our crystals? What is important about having frequency depend only upon the quartz thickness? We answer this with a thought experiment. We start with a crystal that is covered with metallization on both sides. We attach wires and measure it and observe a resonant frequency of 5 MHz. The wires are removed and we snap the round disc in two. The result is a pair of half moon D shapes, but both still have metal. We can attach wires to each of the pieces and measure them. And we discover to our delight that both are still resonant and that the frequency is the same. We can now take one of the D shapes and again break it in two. And again, we experimentally find that the crystal segments are still resonant at the same 5 MHz.

So, what has changed? The new crystals must have some altered properties.

When we snapped the round crystal disc into two equal D shapes, the area of each will have dropped by 2. Hence, assuming a starting diameter much larger than the crystal thickness, the parallel C0 will have dropped by the same factor of 2. Because Cm=C0/220, the motional capacitance will also have dropped by 2, from 10.13 to 5.06 fF. The resonant frequency is unchanged, so the motional inductance must have increased from 0.1 to 0.2 Henry. Lm continues to increase as we reduce the crystal size.

The smaller basic crystals can now be put into a smaller package. They are cheaper to make, for what used to be enough quartz for one crystal unit has now bccome a half dozen. The new mini crystals will have much higher Lm and lower Cm. This makes them much more difficult to use in the design of a crystal filter, especially if the bandwidth is wide. It also makes the mini crystals much harder to move in frequency in a VXO circuit. Both of these changes can be confirmed by inserting the new crystal equivalent circuit (Fig 2) into a circuit analysis.

The mini crystals will also be more fragile and susceptible to damage by high RF current. The new crystals are smaller, so the current must be reduced in proportion, maintaining the same power density. This is probably not critical, but will impact oscillator circuits we might design. Finally, the mini crystals may be more prone to inter-modulation problems if they are in a signal path.

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[2] The same set of equations in the book by Bottom, page 98, includes expressions for both Lm and Cm. If these are used to calculate a resonant frequency, an expression emerges that depends upon thickness, but not crystal area.

Bottom line: It really **does** matter when the crystals don't look like the ones we used to use. Measurements on new crystal types will reveal the new properties. Once the new crystals have been characterized, a new equivalent model can be used. Circuit analysis with the new crystal model will then reveal the impact on circuit performance.