Transistor Models and the Feedback Amplifier

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(Figure captions appear in blue.) (Measurement Data added at end.)

Applied science, and electronic engineering in particular depend upon models. We never attempt to deal with a complete description of a transistor, vacuum tube, or other device. It's just too complicated. Rather, we deal with models. A model is a simplified picture of the actual device, usually steeped in the language of mathematics.

Transistor models range from very simple linear, frequency independent approximations of physical reality to complex nonlinear frequency dependent descriptions. The more refined and elegant models usually form the basis of computer programs for circuit analysis. These models are potentially much more accurate and complete than the simplified models. But there is a price for this accuracy and completeness. A refined *device* model can often get in the way when we try to understand *circuits*. The more useful analysis, and ultimately the more productive design approach treats circuits with the simplest device model that will do the job. Only after the salient circuit behavior is determined with a simple device model is a more refined analysis performed.

The **circuit** we consider here is the so called feedback amplifier used in many RF applications. This circuit begins with a single transistor (or FET) amplifier with two forms of feedback. When negative feedback is applied in two ways, it allows flexibility that is not available with just one. Eventually we will consider a special two stage design.

Device Models

What is the simplest device model that we can use for the circuit analysis? One traditional bipolar transistor model often regarded as the simplest is a current driven current generator. A common emitter amplifier is modeled as a current controlled current generator described by a parameter β (Beta). The current flowing out of the collector is directly proportional to the current flowing in the base. This familiar model is shown below.

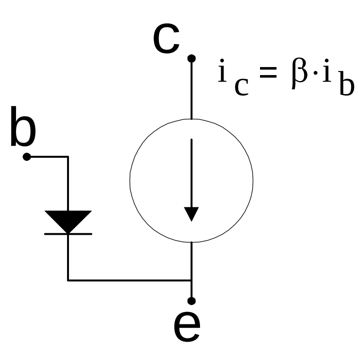
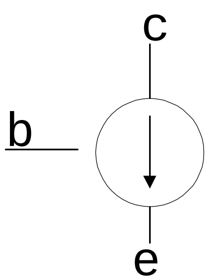


Fig 1. Simple current controlled current generator forms a simple model for the bipolar transistor. This is useful for both small and some large signal calculations including bias evaluation.

This current controlled “Beta Generator” is the model we use for simple bias calculations and sometime for amplifier analysis. But this is not the simplest model, nor is it always the best of the simple models. Voltage driven models are also extremely useful. The transistor is now modeled as a current generator controlled by an input base ***voltage***, shown below.



$$i_c = g_m \cdot v_{be}$$

$$i_c = g_m \cdot (v_b - v_e)$$

Fig 2. This model for the bipolar transistor is a current generator that is controlled by a **voltage**, v_{be}. The “be” in the subscript for this voltage indicates that it is the base voltage measured with respect to the emitter voltage, or $v_{be}=v_b-v_e$.

The voltage controlled model shown above is a **small signal** model. The meaning of the term *small signal* is summarized in the sidebar below.

Small Signal Modeling: Consider the following example circuit:

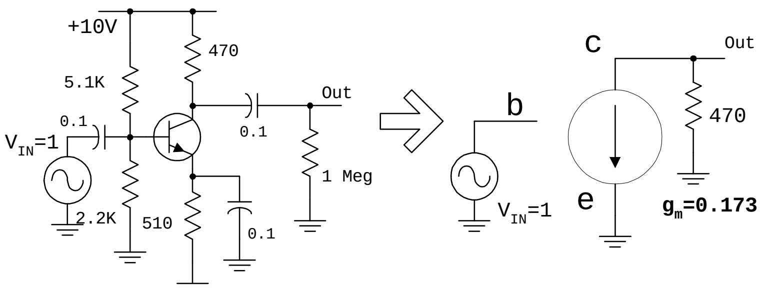


Fig 3. The left schematic shows a circuit with biasing components. The base is driven with a low impedance source, so the impedance of the biasing divider is not important for signal flow. The input impedance of the NPN is of little consequence with a low-Z voltage drive. The emitter circuit contains a 510 Ω bias resistor, but the emitter is bypassed to ground. This merely means that the signal induced variations in emitter current flow in the bypass capacitor and not in the 510 Ω resistor. This circuit is a grounded, or *common* emitter amplifier, for the emitter is common to input and output. The output load is the parallel combination of the 1 Meg and the 470 Ω, which is essentially just 470 Ω. Analysis of the NPN bias shows an emitter current of 4.5 mA. As we will find later, this establishes the transconductance, g_m, for the NPN at 0.173. The resulting small signal circuit is that at the right, a considerable simplification over the original.

Often when performing small signal analysis, voltage and current levels are used that seem far from small. For example, a 1 volt signal applied to the above circuit yields a *small signal* current of 0.17 amp. That output current flows in a 470 Ohm load for a small signal output voltage of, from Ohm's Law, 81 volts. We neglect the collision with the reality of the left circuit and calculate a voltage gain of 81. The signal currents and voltages are both well beyond the bias values, but this can be ignored, for the voltage gain is the detail sought. Kilovolt or microvolt signals both work in a *small signal* model.

The Essence of Emitter Degeneration

The voltage driven model presented in Fig 2 is especially useful when we consider emitter degeneration, which is a resistance in the emitter circuit that is not bypassed. One of the virtues of modeling with simplified elements is that it allows the discovery of circuit behavior that might otherwise be obscure if we tried to do the analysis with more complicated, complete models. Consider the following general circuit.

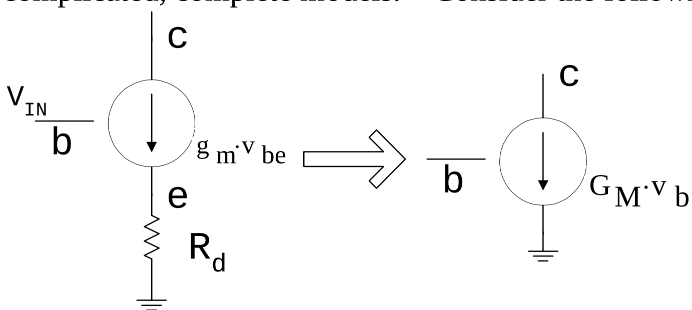


Fig 4. The left current generator is specified by a arbitrary transconductance, g_m. A degeneration resistor R_d is placed in series with the generator. The right model is a simplified circuit with a new, reduced transconductance.

The signal current from the left generator of Fig 4 flows in the degeneration impedance to create an emitter voltage other than ground. The model at the left is analyzed to obtain a new transconductance, G_M, that describes voltage gain for the base with respect to ground. The algebra shows that the new, upper case transconductance is given by the simplified equation

$$G_M = \frac{1}{R_d}$$

This equation applies so long as the original transconductance, g_m, is very large. The mathematical details are in an appendix file, http://w7zoi.net/fba_with_simple_model.pdf Another extremely useful result is that the voltage gain of such a circuit is a simple ratio.

$$G_V = \frac{-R_L}{R_d}$$

where the negative sign indicates an inverting amplifier.

Some Physics

The voltage drive model of Fig 2 can now be extended by evoking a little bit of physics. This can be found in numerous texts with the most notable (read as "my favorite") probably being that of Gray and Meyer, "**Analysis and Design of Analog Integrated Circuits,"** Second Edition, Wiley, 1984. We have learned that the transconductance of a transistor is simply related to an emitter degeneration resistance. Examination of a more detailed model by Ebers and Moll shows that the bipolar transistor is a device with an exponential behavior,

$$I_e = I_{es} \cdot e^{\frac{q \cdot v}{k \cdot T}}$$

This is a large signal model. Manipulation of this equation produces the small signal approximation

$I_e = g_m \cdot v$ where $g_m = \frac{q \cdot I_o}{k \cdot T}$. I_o is now the bias current, q is the electronic charge, k is Boltzman's constant, and T is absolute temperature in Kelvin. This can be reformatted in more familiar terms as

$g_m = \frac{I_e(\text{mA})}{26}$ where I_e is now the bias emitter current in mA. If we interpret this in terms of the small signal voltage drive model of Fig 2, we conclude that the bipolar transistor is model as the following:

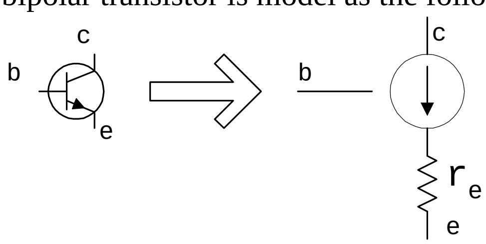


Fig 5. The bipolar transistor is modeled as a voltage controlled current source where the generator has very high transconductance, but is then degenerated with an ***intrinsic emitter resistance***, r_e with value $26/I_e$(mA).

Before applying this model to the feedback amplifier, consider its importance. The model states that **the transistor has a gain that is proportional to the standing bias current**. If the device bias current is increased, the gain will also increase. This is the most fundamental tool available to the designer. The second is feedback.

Total Degeneration

Emitter degeneration has been presented in two forms. One was a completely general r_d. The other was the intrinsic emitter degeneration related to bias. The two are combined below to produce a total degeneration, which we signify with an upper case R_d.

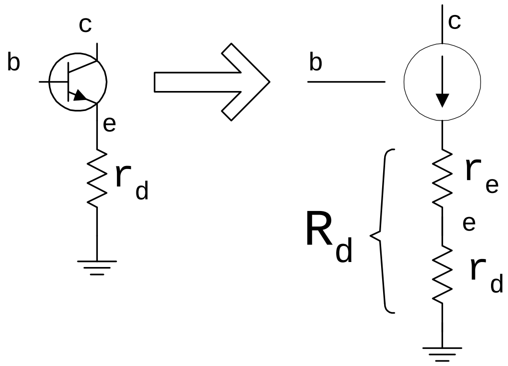


Fig 6. The total degeneration in a common emitter amplifier is R_d consisting of the external degeneration from r_d plus the internal or intrinsic degeneration from r_e.

Why would one degeneration be used over the other? The current dependent r_e is part of the transistor. As such, it can be nonlinear in the large signal equivalent model. This means that it can generate harmonic and intermodulation distortion products in a real world circuit. In contrast, r_d is a simple resistor, a linear element for both small and large signals. The most linear circuits will be those with high bias current (and thus small r_e) in conjunction with enough r_d to constrain the gain to a modest level.

Analyzing the Feedback Amplifier

The familiar feedback amplifier is shown below in small signal form. Bias and other DC details are omitted. This circuit uses two forms of negative feedback: emitter degeneration and parallel collector-to-base feedback.

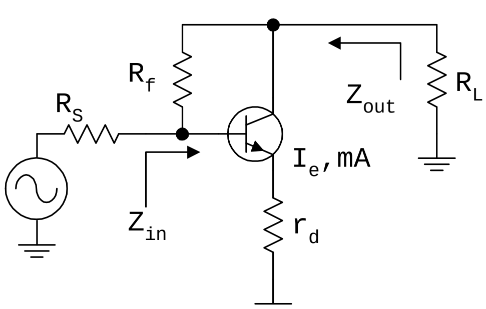


Fig 7. Fundamental feedback amplifier.

The transistor is modeled as an ideal voltage controlled current source with a transconductance that depends on the DC bias current, as discussed above,

$$g_m = \frac{1}{r_e} \quad \text{where} \quad r_e = \frac{26}{I_e(mA)}$$

External degeneration is added, shown as r_d in the above circuit. The two degeneration resistances merge to become a single (upper case) R_d. The simple transistor model is then used to analyze the circuit for gain as well as input and output impedance. Details are given in http://w7zoi.net/fba_with_simple_model.pdf .

Transducer gain is given as

$$G_t = 10 \cdot \log\left[\frac{4 \cdot R_L \cdot R_S \cdot (R_f - R_d)^2}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)^2}\right] \quad \text{dB.}$$

G_t depends upon the feedback elements R_f and R_d as well as the terminations R_S and R_L. Gain depends upon current which is described by r_e, which is a part of R_d.

The input and output impedances are given by

$$Z_{in} = R_d \cdot \frac{(R_L + R_f)}{(R_L + R_d)} \qquad Z_{out} = \frac{R_d \cdot (R_f + R_S)}{(R_d + R_S)}$$

Input impedance depends upon the output load resistance while the output impedance depends upon the source resistance.

An interesting detail that flows from the analysis is that the amplifier is perfectly matched with Z_{in}=R_S and Z_{out}=R_L if the total degeneration R_d is set to $R_d = R_S \cdot R_L / R_f$. However, this relationship is exact only if $R_S = R_L$. It is otherwise just an approximation. Consider the case of equal 50 Ohm source and load resistances and a degeneration resistance defined by the above equation. The following curves then describe the amplifier:

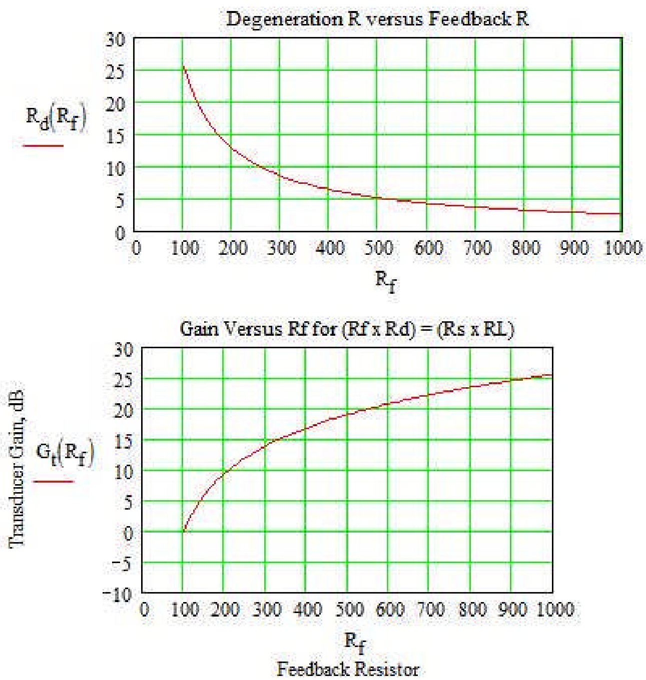


Fig 8. Gain and degeneration resistance versus feedback R for the special case of a 50 Ohm source and 50 Ohm load and $R_f \cdot R_d = R_S \cdot R_L$.

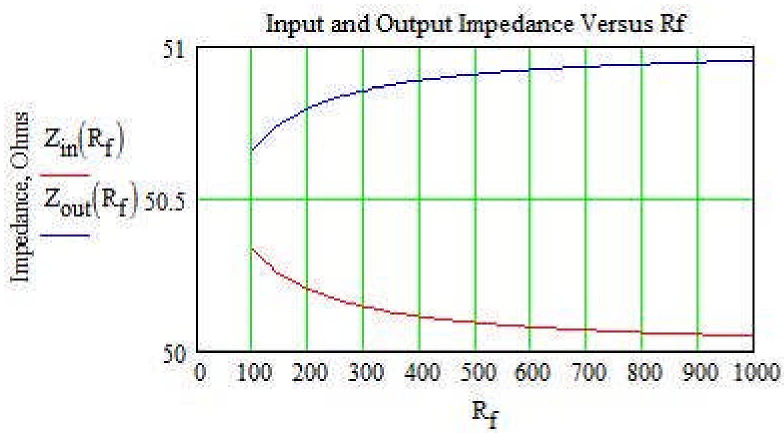
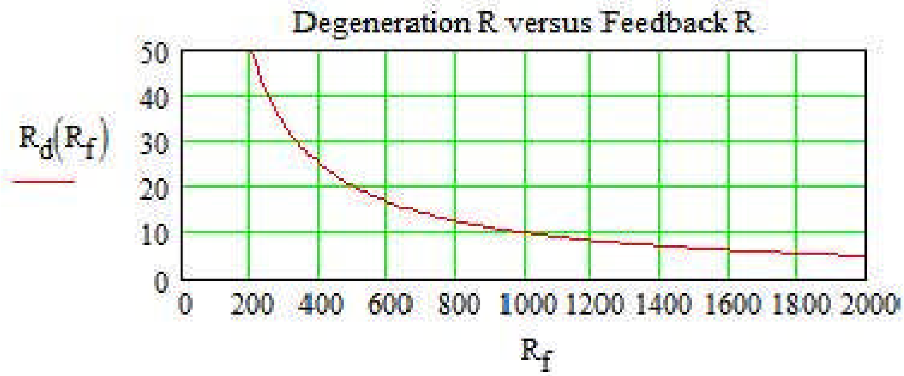
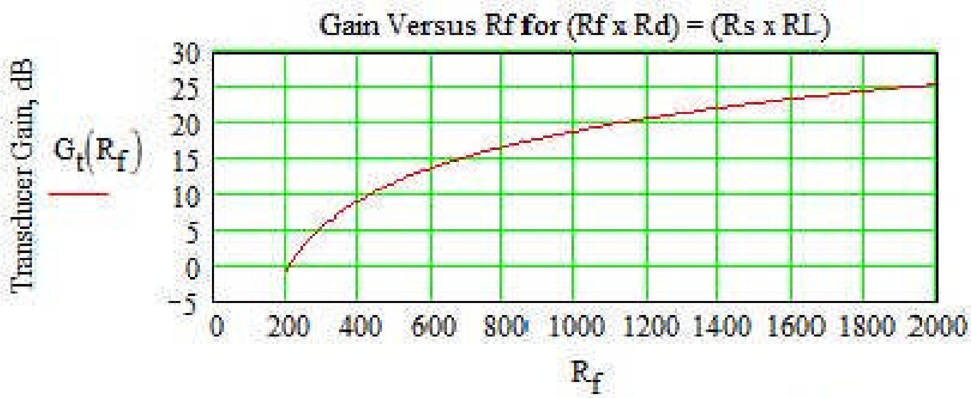


Fig 9. Input and output impedance versus feedback for the "matched" condition, $R_f \cdot R_d = R_S \cdot R_L$. The calculation was done for R_L =51. If it had been exactly 50 like R_S, the two curves would have appeared on top of each other.

The following curves show an amplifier with a 50 Ohm source and a 200 Ohm load. This is a popular configuration that offers better efficiency and intercept when a modest power supply (12 volts) is available. We again apply $R_f \cdot R_d = R_S \cdot R_L$.





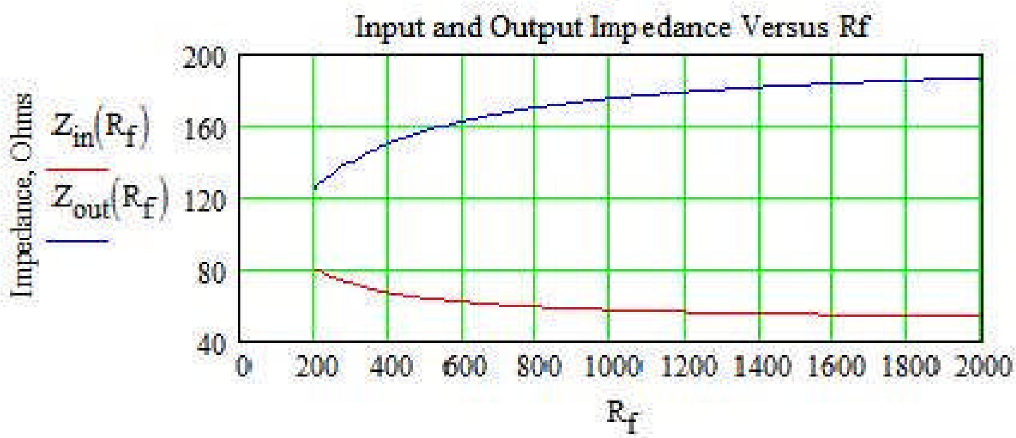


Fig 10. Impedances, degeneration resistance, and transducer gain for the feedback amplifier with 200 Ohm load, 50 Ohm source, and $R_d = R_S \cdot R_L / R_f$.

An amplifier of general interest is the simple bidirectional circuit of EMRFD Fig 6.110. This circuit is shown below, but with amplification in only one direction.

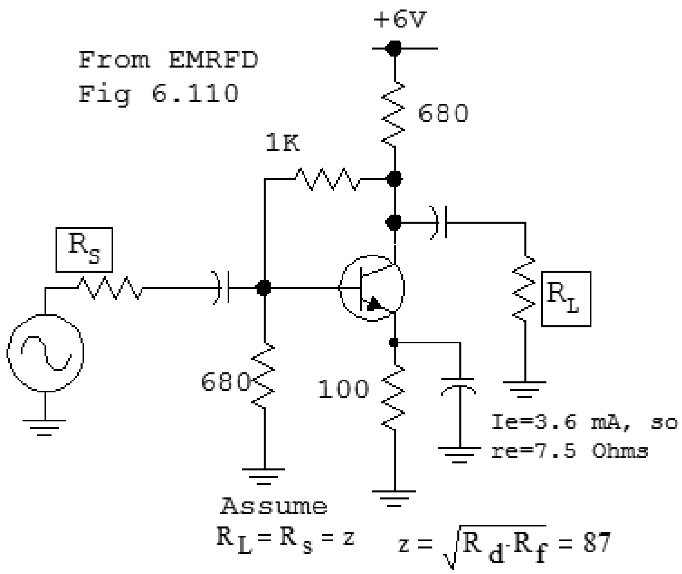


Fig 11. A typical feedback amplifier used in some SSB transceivers described on the web.

This circuit, in one form or another, has been used in some recent web site applications. The amplifier in this figure is biased to an emitter current of only 3.5 mA. As such, r_e is 7.5 Ohms. Because this amplifier uses no external degeneration, R_d is also 7.5 Ohms. The input impedance for any load, or the output impedance for any source can be calculated. If the source and load are forced to have the same value, a *characteristic impedance* can be evaluated for the amplifier, shown in the figure as 87 Ohms. It is clearly not a 50 Ohm circuit even though it is often used with 50 Ohm terminations. It may be a poor termination for filters.

An Enhanced Transceiver Amplifier Block

The ideal amplifier for use in a transceiver is one that has input and output impedances that are independent of the terminations. Yet feedback amplifiers are often used, for they offer stable gain and freedom from self oscillation, which is another form of stability. The virtues of stable impedances and well controlled, stable gain can be realized with a simple circuit if more than one transistor is used. An example circuit is shown below.

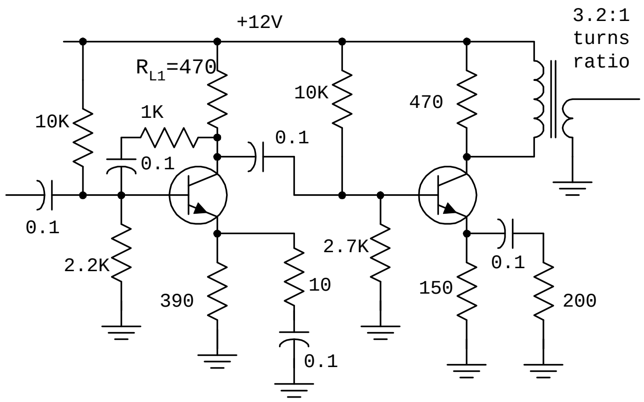


Fig 12. A transceiver gain block. Two stages are cascaded. The first stage has a 470 Ohm load resistance. Parallel feedback and emitter degeneration force the input to 50 Ω. The intermediate load is the source for the second stage. The output transformer is back terminated to generate a 50 Ω output resistance.

The small signal version of this circuit is

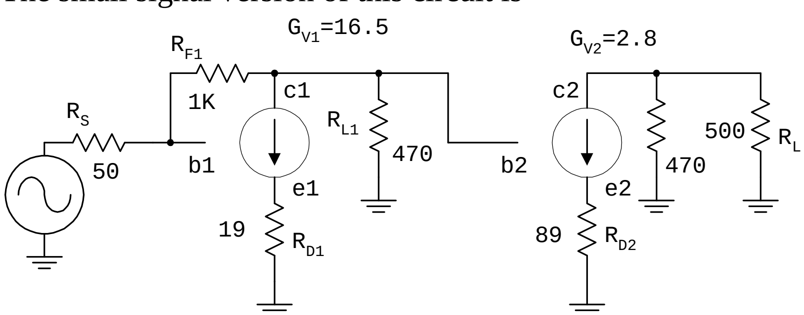


Fig 13. The transceiver gain block in small signal form.

The previous equations are applied to see that the input resistance of the first stage is about 57 Ω with a net voltage gain of 16.5. The gain can be changed by altering the usual feedback elements as well as the load. The second stage contains no parallel feedback, but is still gain stable because of emitter degeneration. The voltage gain is less for this stage. A back termination guarantees a good output match. Analysis with the circuit of Fig 13 shows a transducer gain of 23 dB.

The circuit of Fig 12 was analyzed in LT SPICE using 2N3904 transistors biased at 3 and 10 mA for the two stages. The 10 MHz gain was 23 dB. Input return loss was 19 dB while the output return loss was 26 dB. There was only a slight change in either port when the opposite port was terminated in a 2:1 VSWR. This circuit should offer very stable terminations for filters that are used in two directions in bidirectional SSB transceivers.

The design presented above has not been optimized. As such, it will probably suffer from compromised IMD as well as poor noise figure. It still illustrates the results available when designing with simplified, idealized models.

Measured Results

The simple calculations for the amplifier of Fig 12, followed by more detailed simulations were just too much to ignore. On top of that, Bob Kopski (K3NHI) beat me to the punch by building and measuring one during an east coast rainy spell. The rains arrived here today (June 19th) so I turned the soldering iron on and built one of my own.

The measured results are everything that the calculations said that they would be. The first parameter measured was gain and it came in at 22.5 dB. (All quoted data is at either 10 or 14 MHz. Sweeps go from 1 to 50 MHz.) Forward and reverse gains are shown below.

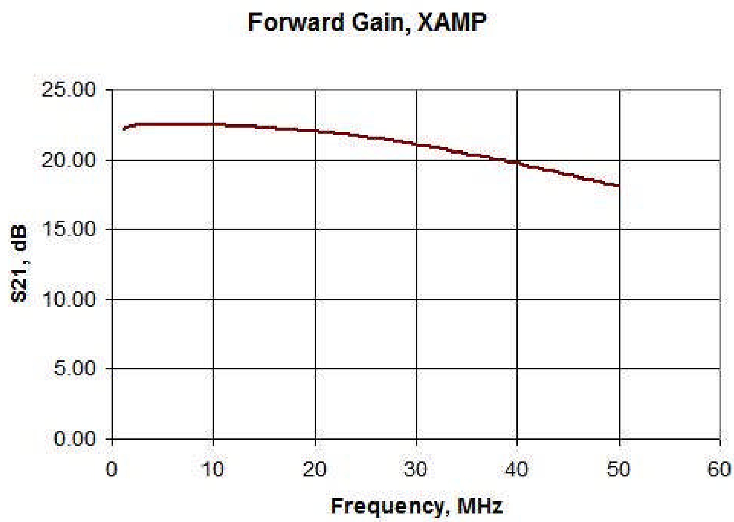


Fig 14. Gain versus F.

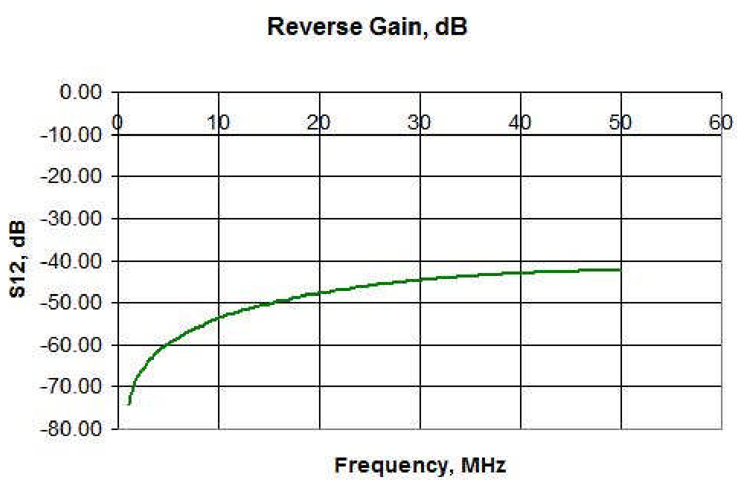


Fig 15. Reverse Gain.

The input match is presented in the next figures.

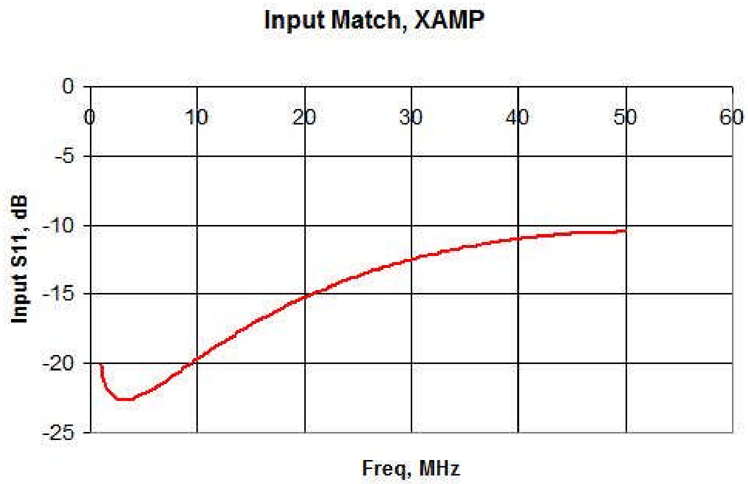


Fig 16. Input impedance match in rectangular form.

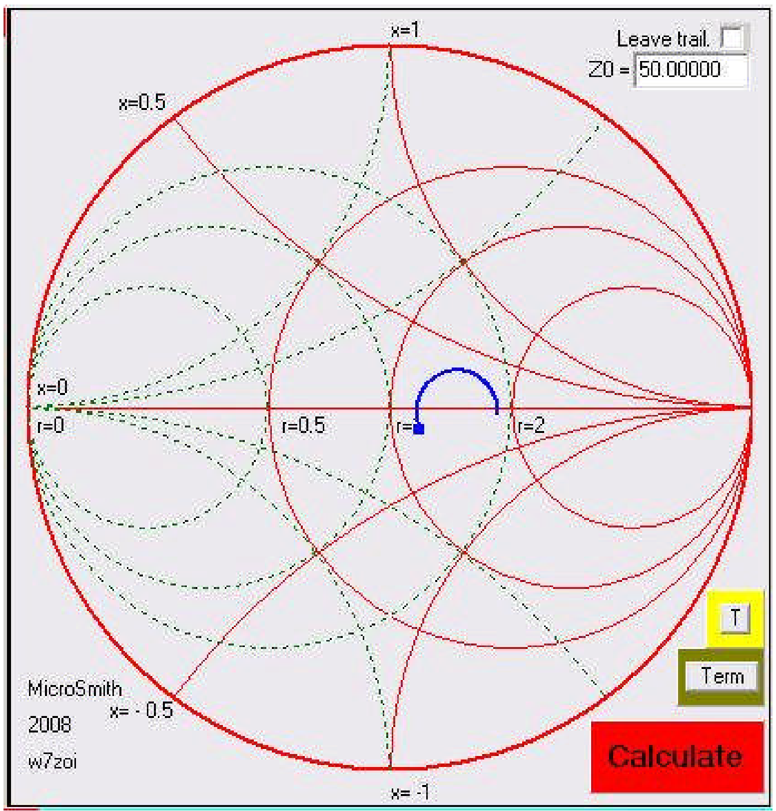


Fig 17. Smith Chart representation of input impedance match from 1 to 50 MHz. Slight adjustment would allow the input to be centered about the chart center.

The next curves show the output match.

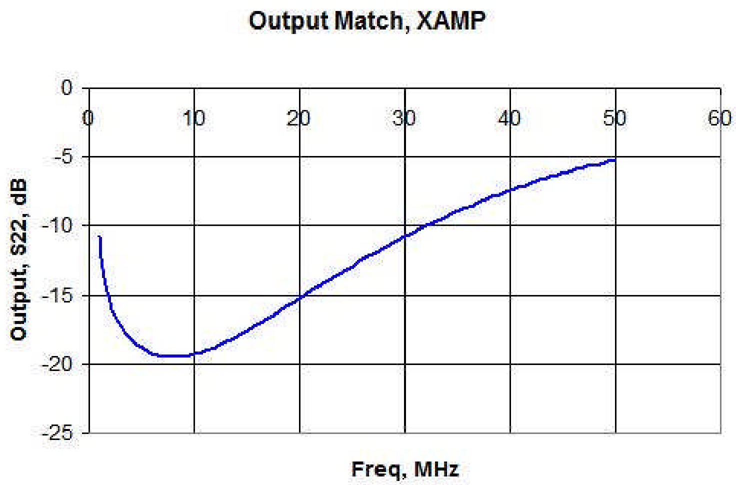


Fig 18. Output impedance match in rectangular form. Separate measurements indicated that the poor wideband match at the upper frequencies resulted from the transformer, which was 16:5 turns on a FB43-2401 toroid.

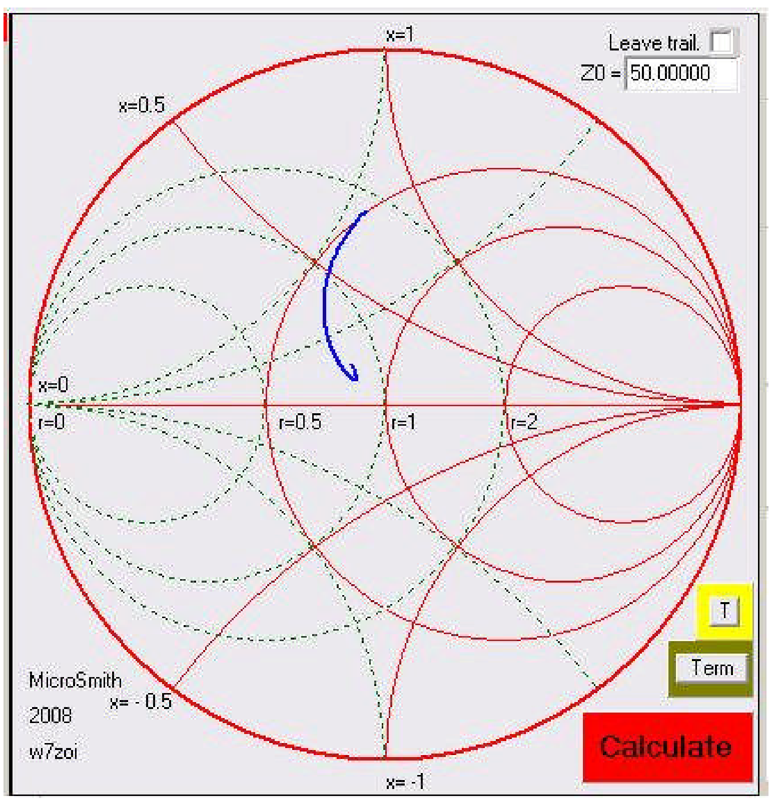


Fig 19. Output impedance match in Smith Chart form. The "hook" corresponds to that in the rectangular plot.

The next figure shows the amplifier hanging on the vector network analyzer (VNA, N2PK type) during gain testing. The circuit was built with "ugly" methods, although I did use SMA connectors to match the VNA.

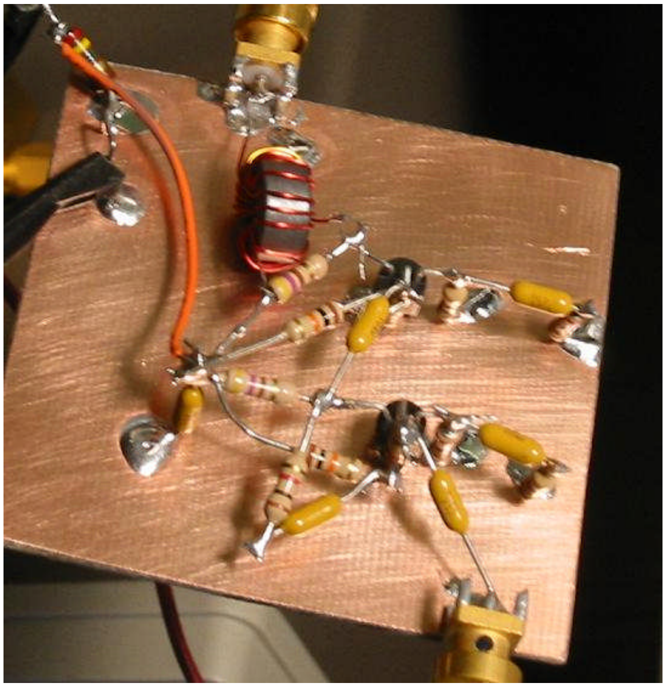


Fig 20. Photo of the amplifier board.

The circuit was tested for noise figure with a Noise Com noise diode source. The result was a pleasant surprise with NF=5 dB. The third order IMD was measured next for the board with results shown below.

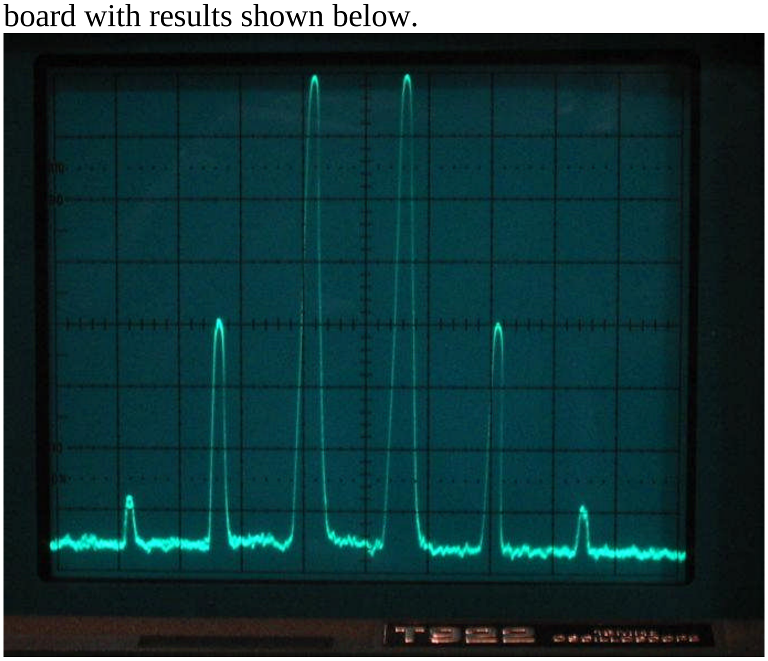


Fig 21. Third order IMD products are 38 dB below the top of the screen at 0 dBm per tone for OIP3=+19 dBm. The IMD was well behaved. This amplifier is not strong enough for use as a post-mixer stage to follow a diode ring in a serious superhet receiver. A little more work might get it there.