Gamma, Return Loss, and VSWR

w7zoi, 18July06

We are always interested in the performance of the circuits we build. If we build an amplifier or a mixer, we are primarily interested in the gain of the circuit. But other parameters also come into play. A detail that is of vital concern with virtually any circuit we build, be it a small signal amplifier for a receiver, a mixer, or even an antenna is the impedance presented by the circuit. An antenna usually has but one impedance of interest, that at the feedpoint. An amplifier will have two "ports" of interest: the input and the output. A mixer is even worse, having three ports. Amplifiers and mixer performance can be complicated because the impedance at one port (e.g., amplifier input) will depend on the load at another port (e.g., amplifier output.) Mixers can be even worse in this regard, for the impedance at one port may depend on the load presented to another port at a different frequency.

It's not a simple world. Our goal here is not to trivialize circuits and the parameters that describe them, but to merely present the relationships in one place. We assume the reader is familiar with the complex numbers normally used when dealing with impedances.

We can specify the ports by the actual impedance that we might "see" when we look into them. Such a measurement could be performed with a vector network analyzer. Alternatively, we can do partial measurements that don't go this far. Rather than measuring the actual impedance, we perform instead a measurement that indicates the quality of the impedance match. In the common situations where we may connect to the amplifier, mixer, or antenna with transmission lines, we measure transmission line parameters. The "*quality of match*" parameters then are indicators of how close an impedance is to a characteristic impedance. Three of these *quality of match* parameters are voltage reflection coefficient, return loss, and VSWR.

It is a value judgment to say that one of these parameters is more fundamental or basic than the other. From my perspective, the most fundamental of the three is ***voltage reflection coefficient***, often signified by upper case Greek ***Gamma***. This is defined for a complex impedance Z and characteristic transmission line impedance Z0 as

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

Consider an example, a complex impedance Z = 77 + j 120 . We assume a characteristic impedance of 50 Ohms. Then

$$\Gamma = \frac{77 - 50 + j \cdot 120}{77 + 50 + j \cdot 120} \quad \text{or} \quad \Gamma = \frac{27 + 120 \cdot j}{127 + 120 \cdot j}$$

We multiply top and bottom by the complex conjugate of the denominator, 127 - j 120. This leaves a denominator that is pure real, or a scalar. The numerator multiplication leaves a real and an imaginary part. These are then divided by the real denominator to obtain Gamma.

$$\Gamma = \frac{(27 + 120 \cdot j) \cdot (127 - j \cdot 120)}{(127 + 120 \cdot j) \cdot (127 - j \cdot 120)} \quad \text{which becomes} \quad \Gamma = .5840 + .3931 \cdot j$$

$$|\Gamma| = \sqrt{.5840^2 + .3931^2} \qquad |\Gamma| = 0.704$$

We have omitted the intermediate steps, leaving them to be completed by the reader.

Gamma is constrained to values with magnitudes between 0 and 1 for impedances with a positive resistance. A circuit impedance with a negative resistance is one capable of oscillation.

The voltage standing wave ratio, ***VSWR,*** is related to ***Gamma*** by

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \text{so,} \quad |\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

VSWR = 5.757 for our example of Z=77+j120.

Return Loss is measured in dB. It is defined in dB by a relationship with the magnitude of Gamma,

$$RL = -20 \cdot \log(|\Gamma|)$$

For the example, the return loss is 3.049 dB. The higher the return loss, the closer VSWR is to 1, and the better the impedance match becomes.

Return Loss is especially useful because it is easily measured with a "Return Loss Bridge," a signal source, and a suitable detector.

The source can be a signal generator. Simple return loss bridges are described in EMRFD, Chapter 7. The signal detector can be a low level power meter such as those using an Analog Devices AD8307 or similar part. (See EMRFD Ch. 7.) Alternatively, a spectrum analyzer, a 50 Ohm terminated oscilloscope, or a receiver calibrated for signal amplitude could all be used. The spectrum analyzer is the preferred detector.

Gamma and VSWR are directly related to return loss. Once one of the three parameters is measured, we know the other two through calculation. If we measure the return loss in dB, we then calculate Gamma and VSWR.

Working backwards,

$$|\Gamma| = 10^{\frac{-RL}{20}} \qquad VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Note that these equations use only the magnitude of the reflection coefficient Gamma. Going back to the early equations that define Gamma for an arbitrary impedance Z, we see that Gamma has both a real and an imaginary part, much like impedance. The magnitude of Gamma is a measure of the quality of an impedance match. Gamma goes to zero, return loss becomes a high dB value, and VSWR is 1 when an impedance approaches Z0.

Those familiar with the Smith Chart will recognize that all three parameters are well represented. The center of the chart is a dot that represents the characteristic impedance, often 50 Ohms. Any point within the "unit circle" of the chart represents a complex impedance. Points above the horizontal line through the center have a positive real part, indicating an inductive reactance. Those on the lower half of the circle are capacitive. The fractional distance from the chart center to a point representing an impedance is Gamma, the reflection coefficient. All impedances with a specified Gamma, VSWR, or return loss will lie on a circle concentric with the chart center. Those wishing to dive into an introduction to the Smith Chart may wish to look at Chapter 4 of Introduction to RF Design. Microsmith including a program manual and other programs intended to accompany IRFD are available as a download from the ARRL web site. Microsmith is a Smith Chart program for the PC. All of these programs are written in DOS, but function with all PC operating systems. This includes XP, although it is difficult to obtain a screen printout with XP.