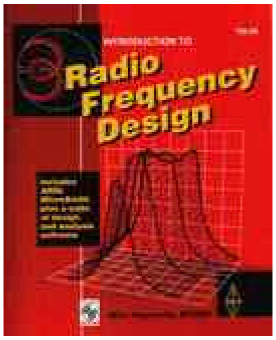
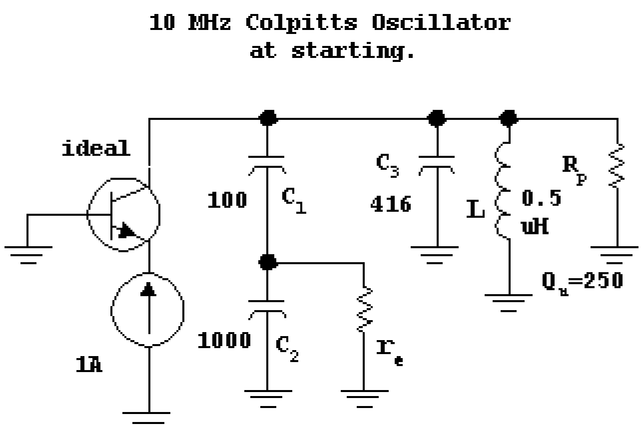


Update Notes for Introduction to Radio Frequency Design

October 31, 2001, returned to web on Nov 9, 2005.

I got an e-mail the other day (that is, in 2001) that pointed out an error in Chapter 7 dealing with oscillators. This had to do with Figure 7.4 which shows the effect of emitter bias current in an NPN Colpitts oscillator on starting gain and phase. The circuit used for analysis is presented here:



In an ideal NPN, the signal current presented to the emitter emerges from the collector. A collector voltage is then generated, defined by the net impedance presented to the collector. Here is where I made an error. In my analysis back in the pre PC era, I assumed that the impedance was resonant and constant. This is not a good assumption as you can show through a more complete analysis. You really have to do the analysis with more care. The resonance is altered by the loading from the emitter current, as transformed through the Colpitts network of C1 and C2.

The next figures show a more up-to-date analysis in MathCad.

The Colpitts collector feeds into several parallel elements (Fig. 7.2b) with one being the Colpitts divider. At the "output," which is the emitter of the closed loop circuit, we have an R and C in parallel. We will start off in the s-domain.

The net admittance presented to the collector is that of the RC plus that of the inductor and the loss resistance of the inductor, plus that of the extra tuning capacitance, C3.
Starting with the emitter r and its parallel cap, C2, we get

$Y = g_e + s \cdot C_2$ so $Z = \frac{1}{g_e + s \cdot C_2}$ where ge is 1/re.

The impedance of C1 is $\frac{1}{s \cdot C_1}$ so the Z of the series combination is

$$Z = \frac{1}{g_e + s \cdot C_2} + \frac{1}{s \cdot C_1}$$

And the resulting admittance is $Y = \left(g_e + s \cdot C_2\right) \cdot s \cdot \frac{C_1}{\left(s \cdot C_1 + g_e + s \cdot C_2\right)}$

We model the finite Q inductor as having a parallel L of QωL=Rp with Rp evaluated only at 10 MHz. We can live with that approximation. And Gp=1/Rp.

The total admittance seen by the collector is then:

$$Y_c = \left(g_e + s \cdot C_2\right) \cdot s \cdot \frac{C_1}{\left(s \cdot C_1 + g_e + s \cdot C_2\right)} + s \cdot C_3 + \frac{1}{s \cdot L} + G_p$$

The Colpitts network of C1, C2 and re have a voltage transfer function resulting from simple voltage divider action. The voltage transfer function from collector to emitter is H:

$$H = s \cdot \frac{C_1}{\left(s \cdot C_1 + g_e + s \cdot C_2\right)}$$

We now inject a 1 amp current into the emitter (Fig. 7.2b) and assume that the same current emerges from the collector. The collector voltage generated is Vc,

$V_c = \frac{1}{Y_c}$ and the current flowing in the load is $I = \frac{H}{Y_c \cdot r_e}$ which is just the current gain desired.

We eventually want to see behavior with emitter bias current, so we substitute 26/i for re.

e is the emitter current.

Some of the parameters: $C_1 := 10^{-10}$ $C_2 := 10^{-9}$ $C_3 := 416 \cdot 10^{-12}$

$\omega := 2 \cdot \pi \cdot 10^7$ $L := 0.5 \cdot 10^{-6}$ $G_p := 1.273 \cdot 10^{-4}$

$e := .01, .015 .. 10$

$a := 57.296$ degrees per radian

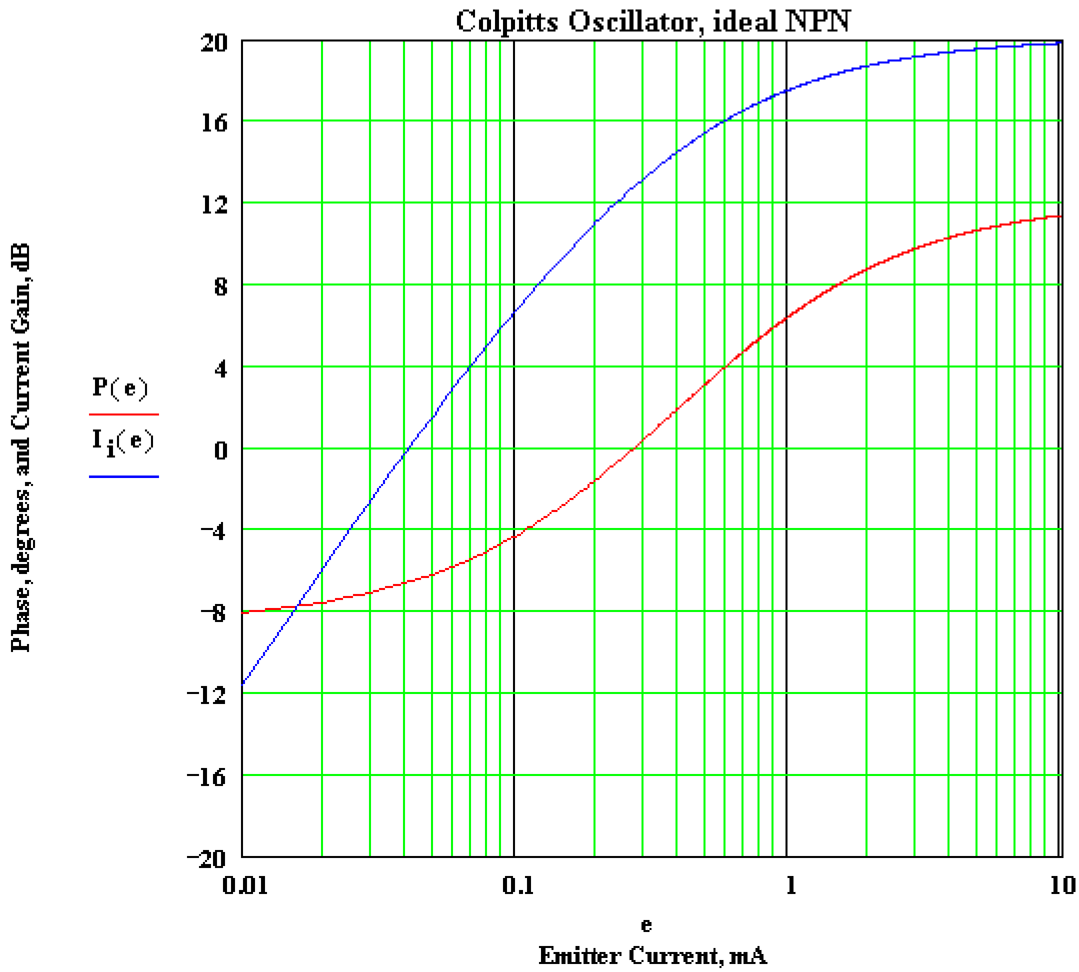
$$H(e) := i \cdot \omega \cdot \frac{C_1}{\left(i \cdot \omega \cdot C_1 + \frac{1}{26} \cdot e + i \cdot \omega \cdot C_2\right)}$$

$$Y_c(e) := i \cdot \left(\frac{e}{26} + i \cdot \omega \cdot C_2\right) \cdot \omega \cdot \frac{C_1}{\left(i \cdot \omega \cdot C_1 + \frac{e}{26} + i \cdot \omega \cdot C_2\right)} + i \cdot \omega \cdot C_3 - \frac{i}{(\omega \cdot L)} + G_p$$

So, current gain is $G_i(e) := \dfrac{H(e) \cdot e}{Y_c(e) \cdot 26}$

The in-phase component of the gain is $I_i(e) := 20 \cdot \log\left(\mathrm{Re}\left(G_i(e)\right)\right)$ dB

And phase is $P(e) := a \cdot \arg\left(G_i(e)\right)$



The general character of the curve is unchanged, but the values are different. Specifically, the phase goes through zero degrees at a bias current of about 0.3 mA.

Continuing the analysis by performing frequency sweeps is interesting. Clearly, our assumptions were wrong with the first analysis. This is easily done with PSPICE or similar simulator.

Many thanks to John Broderick from RF Integration Inc. in Lowell, MA, the sharp eyed reader who caught the error.