The Feedback Amplifier with a Simple Model

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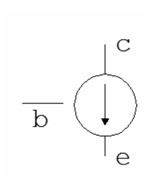
(Figure captions are in blue.)

Equations Derived

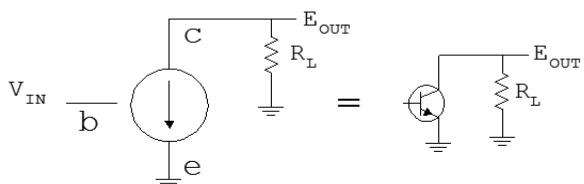
This derivation starts with a transistor described as a voltage controlled current generator with very high transconductance, g_m. Reality as well as current dependence are retained by inserting emitter degeneration.

Assume a voltage controlled current source with very high transconductance g_m. The controlling voltage is that between the base and emitter. The collector current is just

$$i_c = g_m \cdot V_{be} \quad \text{where} \quad V_{be} = V_b - V_e$$



Basic Model



A common emitter amplifier with this model.

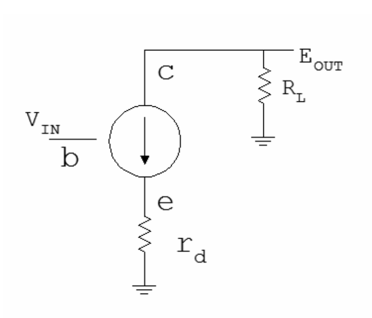
The current in the CE collector flows through the load. Taking note of the polarity indicated by the arrow, the voltage at the collector is

$$E_{out} = -g_m \cdot V_{be} \cdot R_L$$

The voltage gain is then

$$G_v = \frac{E_{out}}{V_{be}} = -g_m \cdot R_L$$

Adding Emitter Degeneration



Common emitter amplifier with degeneration r_d.

Note that V_{in} does ***not*** equal V_{be}. Expanding the earlier equation,

$$i = g_m \cdot (V_b - V_e) = g_m \cdot V_b - g_m \cdot V_e$$

But

$V_e = i \cdot r_d$

(without inversion). Substituting this,

$$V_e = i \cdot r_d$$
$$i \cdot (1 + g_m \cdot r_d) = g_m \cdot V_b$$
$$i = \frac{g_m \cdot V_b}{(1 + g_m \cdot r_d)}$$

Calculate the transconductance of this overall cell, which we define as upper case G_m.

$$i = G_m \cdot V_b$$

$$G_m = \frac{g_m}{1 + g_m \cdot r_d}$$ Dividing top and bottom by g_m,

$$G_m = \frac{1}{\frac{1}{g_m} + r_d}$$

The limit of this expression as g_m becomes very large is

$$G_m = \frac{1}{r_d}$$

Extending the earlier analysis, the amplifier voltage gain becomes

$$G_v = -G_m \cdot R_L = \frac{-R_L}{r_d}$$

The device transconductance does not really matter so long as g_m is large, ($g_m >> 1/r_d$).

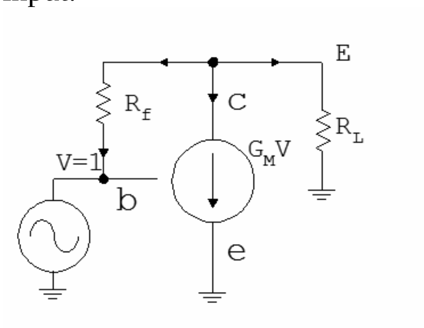
Current Dependence

Recall the parameter $r_e = 26/I_e$ from the basic physics of the junction transistor. I_e here is emitter current in mA. The simplest small signal model for a bipolar transistor is then an infinite transconductance amplifier that is then degenerated by an intrinsic resistance r_e. Beta (β) does not appear as a parameter and the circuit is *voltage* driven. This does not imply that no current flows in the input, for it does and is related to collector current with β. Rather, this is just the simple model used in this discussion. Voltage drive describes many of the salient properties of the bipolar transistor, making this the generally accepted, simplest level bipolar transistor model.

The degeneration in the above equations is described by r_d, which is external to the current source, but we then introduce r_e as a degeneration that describes the intrinsic transistor. In practice, we usually use a combination, upper case R_d, which is just the sum of the internal and external degenerations, r_e and r_d.

Adding Parallel Feedback

Degeneration is one form of feedback. Consider now an amplifier with a small signal transconductance G_m. We use the upper case G_m in this case to emphasize that we may well have a high gain transistor that is degenerated by a resistance. We introduce parallel feedback as a resistor from the output terminals back to the input. The arrows show assumed directions of current used in forming equations. A 1 volt source drives the input.



Amplifier with parallel feedback. Degeneration may also be present.

All current is out of the collector node, so

$$0 = G_m + \frac{(E-1)}{R_f} + \frac{E}{R_L}$$

Solving this for the voltage "E" yields

$$E = \frac{-(G_m \cdot R_f - 1)}{(R_L + R_f)} \cdot R_L$$

Voltage gain is defined as

$$G_V = \frac{E}{V}$$

and V=1, so

$$G_V = \frac{-(G_m \cdot R_f - 1)}{(R_L + R_f)} \cdot R_L$$

We now ask for the input impedance. This is calculated from the input voltage, 1, and the *input current*.

$$i_{in} = \frac{E-V}{R_f} = \frac{E-1}{R_f} \qquad Z_{in} = \frac{1}{i_{in}} = \frac{R_f}{E-1}$$

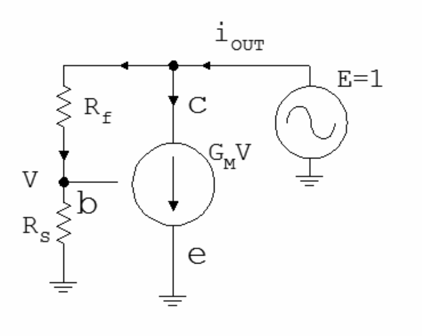
Substitute E from the above analysis to find Z_{in}.

$$Z_{in} = \frac{R_f}{E-1} = \frac{R_f}{\frac{-(G_m \cdot R_f - 1)}{(R_L + R_f)} \cdot R_L - 1} = \frac{-(R_L + R_f)}{(R_L \cdot G_m + 1)}$$

This impedance is negative, but that is merely the result of the arbitrary current direction assumed above. Change the sign to fit with current into the amplifier from the external source to obtain

$$Z_{in} = \frac{(R_L + R_f)}{R_L \cdot G_m + 1}$$

Next, we wish to calculate the output impedance. The ***output*** is driven with a 1 volt generator while terminating the input in a source resistance R_S. The input voltage at the base is then obtained by voltage divider action.



Setup for calculation of output impedance.

$$V = \frac{E \cdot R_S}{R_S + R_f}$$

This allows calculation of collector current.

$$i_c = G_m \cdot V = G_m \cdot \frac{E \cdot R_S}{R_S + R_f}$$

The current into the collector node, i_{out}, is then obtained. One current flows into the collector node and two flow out of the node. The nodal equation is

$$i_{out} = \frac{E}{R_f + R_S} + i_c = \frac{1}{R_f + R_S} + G_m \cdot \frac{R_S}{R_S + R_f} = \frac{(1 + G_m \cdot R_S)}{R_f + R_S}$$

The output impedance, which is just the Z looking into the output port, becomes

$$Z_{out} = \frac{E}{i_{out}} = \frac{R_f + R_S}{(1 + G_m \cdot R_S)}$$

Rather than using G_m for our calculations, we will now express it in terms of degeneration. From the early analysis,

$$G_m = \frac{1}{R_d} \qquad \text{and} \qquad R_d = r_d + r_e$$

An upper case R_d is used to emphasize that it could and often is the sum of an external degeneration and the internal r_e. Manipulation yields

$$G_v = \frac{-R_L \cdot (R_f - R_d)}{R_d \cdot (R_L + R_f)} \qquad Z_{in} = R_d \cdot \frac{(R_L + R_f)}{(R_L + R_d)} \qquad Z_{out} = \frac{R_d \cdot (R_f + R_S)}{(R_d + R_S)}$$

Matched Impedances

What are the conditions for an impedance match? At the input port

$$R_S = R_d \cdot \frac{(R_L + R_f)}{(R_L + R_d)}$$

Solving this for the degeneration resistance R_d,

$$R_d = R_S \cdot \frac{R_L}{(-R_S + R_L + R_f)}$$

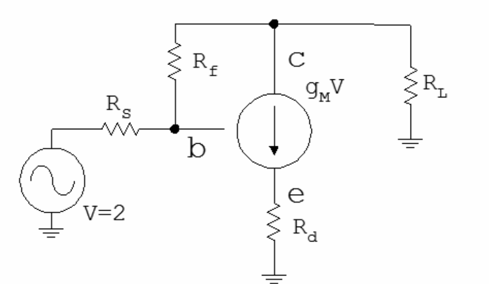
This defines the parameters that will produce an input match for any arbitrary load, source, and feedback resistor. If $R_S = R_L$ then this collapses to the familiar form:

$$R_d \cdot R_f = R_S \cdot R_L$$

A similar analysis at the output port will produce the same equation that defines that match, so $R_d\ R_f = R_S\ R_L$ defines a simultaneous match at both ports, for the special case of $R_S = R_L$. This relationship is otherwise just an approximation.

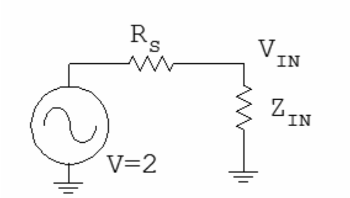
Transducer Gain

We now calculate the **transducer gain** of the amplifier, as shown in this circuit.



Circuit used to calculate G_t which is defined as the output power divided by the power ***available*** from the source. The driving source is a generator with a 2 volt open circuit strength.

The driving generator has an open circuit strength of 2 volts. If a 2 volt source with source impedance R is terminated in an equal R, the result will be 1 volt across that impedance. This leads to straight forward calculations of available power, $1/R_S$. This is detailed in the figure below.



The input base signal is calculated by knowing Z_{in} and R_s.

The input impedance was found earlier. $Z_{in} = R_d \cdot \frac{(R_L + R_f)}{(R_L + R_d)}$

The 2 volt source will be divided by this and the source.

$$V_{in} = \frac{2 \cdot Z_{in}}{Z_{in} + R_S}$$

which becomes

$$V_{in} = 2 \cdot R_d \cdot \frac{(R_L + R_f)}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)}$$

This signal is amplified by the voltage gain.

$$G_v = \frac{-R_L \cdot (R_f - R_d)}{R_d \cdot (R_L + R_f)}$$

$$E = G_v \cdot V_{in} = \frac{-R_L \cdot (R_f - R_d)}{R_d \cdot (R_L + R_f)} \cdot \left[2 \cdot R_d \cdot \frac{(R_L + R_f)}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)} \right]$$

$$E = G_v \cdot V_{in} = -2 \cdot R_L \cdot \frac{(R_f - R_d)}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)}$$

Calculate the output power.

$$P_{out} = \frac{E^2}{R_L} = \frac{\left[-2 \cdot R_L \cdot \frac{(R_f - R_d)}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)} \right]^2}{R_L}$$

$$P_{out} = \frac{E^2}{R_L} = 4 \cdot R_L \cdot \frac{(R_f - R_d)^2}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)^2}$$

The available power from the source is

$$P_{av} = \frac{1}{R_S}$$

Hence, transducer gain is

$$G_t = \frac{P_{out}}{P_{av}} = \frac{4 \cdot R_L \cdot R_S \cdot (R_f - R_d)^2}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)^2}$$

This is usually expressed in dB form rather than as a simple power ratio,

$$G_t = 10 \cdot \log\left[\frac{4 \cdot R_L \cdot R_S \cdot \left(R_f - R_d\right)^2}{\left(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d\right)^2}\right] \qquad \text{dB.}$$