The Feedback Amplifier with a Simple Model
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(Figure captions are in blue.)

Equations Derived

This derivation starts with a transistor described as a voltage controlled current generator with very high transconductance, \( g_m \). Reality as well as current dependence are retained by inserting emitter degeneration.

Assume a voltage controlled current source with very high transconductance \( g_m \). The controlling voltage is that between the base and emitter. The collector current is just
\[
I_C = g_m V_{be}
\]
where \( V_{be} = V_b - V_e \)

![Basic Model](Image)

A common emitter amplifier with this model.

The current in the CE collector flows through the load. Taking note of the polarity indicated by the arrow, the voltage at the collector is
\[
E_{out} = -g_m V_{be} R_L
\]
The voltage gain is then
\[
G_v = \frac{E_{out}}{V_{be}} = -g_m R_L
\]

Adding Emitter Degeneration

![Common emitter amplifier with degeneration](Image)

Common emitter amplifier with degeneration \( r_d \).

Note that \( V_{in} \) does not equal \( V_{be} \). Expanding the earlier equation,
\[
i = g_m (V_b - V_e) = g_m V_b - g_m V_e
\]

But
\[V_e = i r_d\]
(without inversion). Substituting this,
\[
V_e = i r_d \\
i (1 + g_m r_d) = g_m V_b \\
i = \frac{g_m V_b}{1 + g_m r_d}
\]

Calculate the transconductance of this overall cell, which we define as upper case \(G_m\).
\[
i = G_m V_b \\
G_m = \frac{g_m}{1 + g_m r_d}
\]
Dividing top and bottom by \(g_m\)
\[
G_m = \frac{1}{\frac{1}{g_m} + r_d}
\]

The limit of this expression as \(g_m\) becomes very large is
\[
G_m = \frac{1}{r_d}
\]
Extending the earlier analysis, the amplifier voltage gain becomes
\[
G_v = -G_m R_L = -\frac{R_L}{r_d}
\]

The device transconductance does not really matter so long as \(g_m\) is large, \((g_m \gg 1/r_d)\).

**Current Dependence**

Recall the parameter \(r_e = 26/I_e\) from the basic physics of the junction transistor. \(I_e\) here is emitter current in mA. The simplest small signal model for a bipolar transistor is then an infinite transconductance amplifier that is then degenerated by an intrinsic resistance \(r_e\). Beta (\(\beta\)) does not appear as a parameter and the circuit is voltage driven. This does not imply that no current flows in the input, for it does and is related to collector current with \(\beta\). Rather, this is just the simple model used in this discussion. Voltage drive describes many of the salient properties of the bipolar transistor, making this the generally accepted, simplest level bipolar transistor model.

The degeneration in the above equations is described by \(r_d\), which is external to the current source, but we then introduce \(r_e\) as a degeneration that describes the intrinsic
transistor. In practice, we usually use a combination, upper case $R_d$, which is just the sum of the internal and external degenerations, $r_c$ and $r_d$.

**Adding Parallel Feedback**

Degeneration is one form of feedback. Consider now an amplifier with a small signal transconductance $G_m$. We use the upper case $G_m$ in this case to emphasize that we may well have a high gain transistor that is degenerated by a resistance. We introduce parallel feedback as a resistor from the output terminals back to the input. The arrows show assumed directions of current used in forming equations. A 1 volt source drives the input.

![Amplifier with parallel feedback. Degeneration may also be present.](image)

All current is out of the collector node, so

$$0 = G_m + \frac{(E - 1) + E}{R_f}$$

Solving this for the voltage “E” yields

$$E = \frac{(G_m R_f - 1)}{(R_L + R_f)} R_L$$

Voltage gain is defined as

$$G_v = \frac{E}{V}$$

and $V=1$, so

$$G_v = \frac{-(G_m R_f - 1)}{(R_L + R_f)} R_L$$

We now ask for the input impedance. This is calculated from the input voltage, 1, and the input current.

$$i_{in} = \frac{E - V}{R_f} = \frac{E - 1}{R_f}$$

$$Z_{in} = \frac{1}{i_{in}} = \frac{R_f}{E - 1}$$

Substitute $E$ from the above analysis to find $Z_{in'}$.
This impedance is negative, but that is merely the result of the arbitrary current direction assumed above. Change the sign to fit with current into the amplifier from the external source to obtain

\[ Z_{\text{in}} = \frac{(R_L + R_f)}{R_L G_m + 1} \]

Next, we wish to calculate the output impedance. The output is driven with a 1 volt generator while terminating the input in a source resistance \( R_S \). The input voltage at the base is then obtained by voltage divider action.

\[ V = \frac{E R_S}{R_S + R_f} \]

This allows calculation of collector current.

\[ i_c = G_m V = G_m \frac{E R_S}{R_S + R_f} \]

The current into the collector node, \( i_{\text{out}} \), is then obtained. One current flows into the collector node and two flow out of the node. The nodal equation is

\[ i_{\text{out}} = \frac{E}{R_f + R_S} + i_c = \frac{1}{R_f + R_S} + \frac{G_m}{R_S + R_f} = \frac{1 + G_m R_S}{R_f + R_S} \]

The output impedance, which is just the \( Z \) looking into the output port, becomes

\[ Z_{\text{out}} = \frac{E}{i_{\text{out}}} = \frac{R_f + R_S}{(1 + G_m R_S)} \]

Rather than using \( G_m \) for our calculations, we will now express it in terms of degeneration. From the early analysis,

\[ G_m = \frac{1}{R_d} \quad \text{and} \quad R_d = r_d + r_e \]

An upper case \( R_d \) is used to emphasize that it could and often is the sum of an external degeneration and the internal \( r_e \). Manipulation yields

\[ G_v = \frac{-R_L (R_f - R_d)}{R_d (R_L + R_f)} \quad \text{and} \quad Z_{\text{in}} = R_d \frac{(R_L + R_f)}{R_L + R_f} \quad Z_{\text{out}} = R_d \frac{(R_f + R_S)}{(R_f + R_S)} \]

\[ Z_{\text{in}} = \frac{R_f}{E - 1} = \frac{R_f}{-\left(C_m R_f - 1\right) - R_L - 1} = \frac{-\left(R_L + R_f\right)}{(R_f C_m + 1)} \]
**Matched Impedances**

What are the conditions for an impedance match? At the input port

\[ R_S = R_d \frac{(R_L + R_f)}{(R_L + R_d)} \]

Solving this for the degeneration resistance \( R_d \),

\[ R_d = R_S \frac{R_L}{(R_S + R_L + R_f)} \]

This defines the parameters that will produce an input match for any arbitrary load, source, and feedback resistor. If \( R_S = R_L \) then this collapses to the familiar form:

\[ R_d \cdot R_f = R_S \cdot R_L \]

A similar analysis at the output port will produce the same equation that defines that match, so \( R_d \cdot R_f = R_S \cdot R_L \) defines a simultaneous match at both ports, for the special case of \( R_S = R_L \). This relationship is otherwise just an approximation.

**Transducer Gain**

We now calculate the transducer gain of the amplifier, as shown in this circuit.

![Circuit used to calculate Gt which is defined as the output power divided by the power available from the source. The driving source is a generator with a 2 volt open circuit strength.](image)

The driving generator has an open circuit strength of 2 volts. If a 2 volt source with source impedance \( R \) is terminated in an equal \( R \), the result will be 1 volt across that impedance. This leads to straightforward calculations of available power, \( 1/R_S \). This is detailed in the figure below.

![The input base signal is calculated by knowing \( Z_{in} \) and \( R_s \).](image)
The input impedance was found earlier.

The 2 volt source will be divided by this and the source.

\[ V_{in} = \frac{2Z_{in}}{Z_{in} + R_S} \]

which becomes

\[ V_{in} = 2R_d \frac{(R_L + R_f)}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)} \]

This signal is amplified by the voltage gain.

\[ G_v = \frac{-R_L \cdot (R_f - R_d)}{R_d \cdot (R_L + R_f)} \]

\[ E = G_v \cdot V_{in} = \frac{-R_L \cdot (R_f - R_d)}{R_d \cdot (R_L + R_f)} \left[ 2R_d \frac{(R_L + R_f)}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)} \right] \]

\[ E = G_v \cdot V_{in} = -2R_L \frac{(R_f - R_d)}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)} \]

Calculate the output power.

\[ P_{out} = \frac{E^2}{R_L} = \frac{-2R_L \cdot (R_f - R_d)^2}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)^2} \]

\[ P_{out} = \frac{E^2}{R_L} = 4R_L \frac{(R_f - R_d)^2}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)^2} \]

The available power from the source is

\[ P_{av} = \frac{1}{R_S} \]

Hence, transducer gain is

\[ G_t = \frac{P_{out}}{P_{av}} = \frac{4R_L \cdot R_S \cdot (R_f - R_d)^2}{(R_d \cdot R_L + R_d \cdot R_f + R_S \cdot R_L + R_S \cdot R_d)^2} \]

This is usually expressed in dB form rather than as a simple power ratio,
\[
G_t = 10 \log \frac{4R_L R_S (R_f - R_d)^2}{(R_d R_L + R_d R_f + R_S R_L + R_S R_d)^2} \quad \text{dB.}
\]