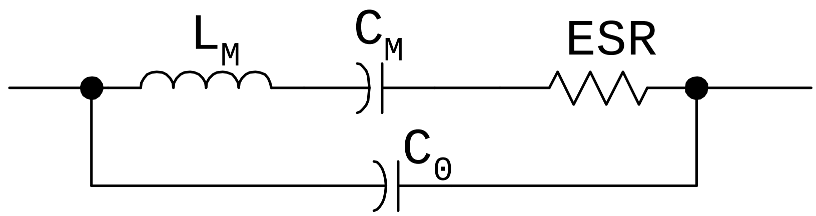
An Oscillator Scheme for Quartz Crystal Characterization.

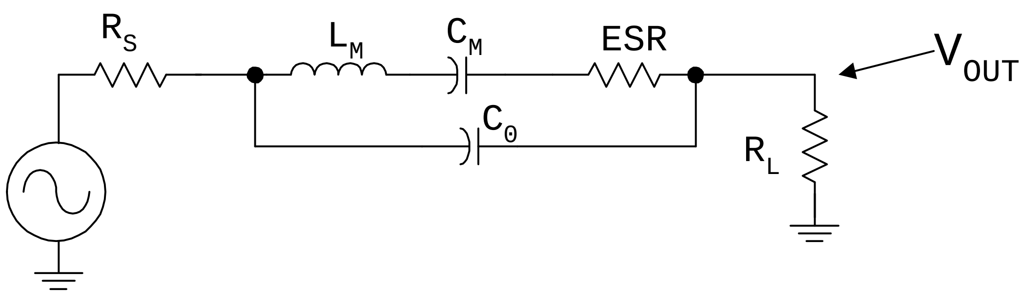
Wes Hayward, 15Nov07

The familiar quartz crystal is modeled with the circuit shown below containing a series inductor L_M, capacitor C_M, and equivalent series resistor ESR, all paralleled by a capacitor C_0. The subscript "M" used with L and C signify that these are *motional* parameters.



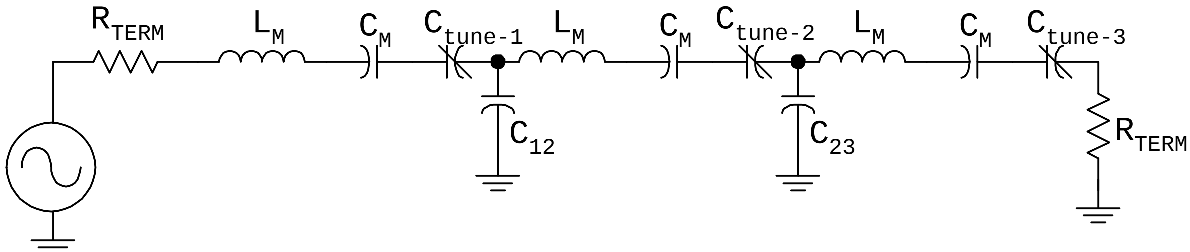
This is a two terminal device, something that has but a single port. The element can be characterized by studying it with a network analyzer where it is placed between a generator and a load. The scattering parameters are then measured. Detailed analysis is then used to extract the four parameters. Alas, this is not possible for those of us without a basement network analyzer and it is not very intuitive.

One can do a simplified analysis with a stable oscillator (usually a VXO) operating as a signal generator with a very low source impedance and a very low load resistance. This is shown below.



The low source impedance has traditionally been generated with a transformer dropping from 50 Ohms down to a source impedance from 3 to 12 Ohms. Pads are inserted to further establish the impedance. Because the impedance is very low, the parallel capacitance has little impact upon the voltage across the load and can be ignored, at least for this simple analysis, leaving nothing but a series tuned circuit. The generator is tuned to series resonance to produce a maximum output. Comparing this against a piece of wire (a "through connection" to the network analyzer folks) allows the ESR to be inferred from the insertion loss. If the generator is then tuned to one side and then to the other to the – 3 dB points, one can measure a bandwidth. A loaded Q can then be calculated. Comparison of this value with the inferred ESR plus the source and load resistance allows the motional inductance to be calculated. This and the series resonant frequency yield the motional capacitance. Parallel capacitance, C0, can be measured at a low frequency well removed from an resonance with a simple capacitance bridge. I presented some experiments using these methods in a 1982 QST. (See "A Unified Approach to the Design of Crystal Ladder Filters," QST, May, 1982.) The scheme has been used in many others who have been building their own crystal filters.

Bandpass filters can be designed with relative ease using traditional methods if we assume the crystals to be nothing more than series tuned circuits. A filter then takes on the form shown below where the *variable* capacitors are usually just fixed elements inserted to move all meshes to the same frequency, the filter center.



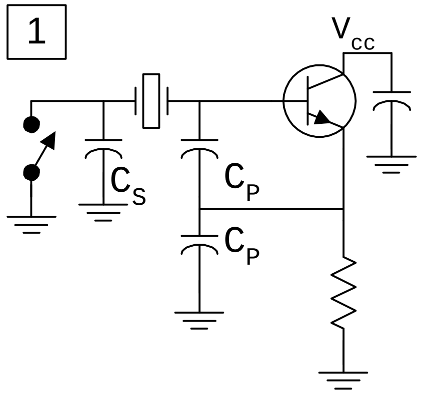
This filter uses three sections. Knowing the motional capacitance allows one to calculate the coupling capacitors C_{12} and C_{23} for a specified bandwidth, using a normalized coupling coefficient for the filter response shape (Butterworth, Chebyshev, etc) that is desired. Similarly, an end section Q can be calculated from a normalized end section Q and the desired bandwidth. Knowledge of this Q and the motional inductance allows one to pick termination values that will yield the termination resistance.

Alas, things are not as simple as we might like. The existence of C_0 with our crystals imposes restrictions. One is limited to narrow filters if using the so called ladder topology depicted. The bandwidth is of the order of the difference between the series and parallel resonant frequencies of the crystals. Even when building filters within the allowed bandwidth limits, the parallel capacitance complicates the process.

Crystal Characterization with an Oscillator

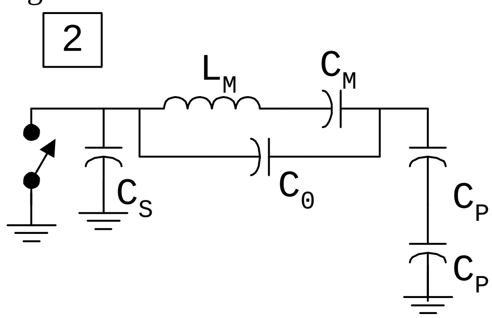
Crystal ladder filters can be designed if one knows the motional parameters. Although the network analyzer schemes are ideal, one can also do a good job with a crystal oscillator. Such a circuit is shown in Fig 3.35 on page 3.19 of Experimental Methods in RF Design. The circuit is presented in EMRFD with a simple equation for motional C, but nothing is offered in the way of a hint about where the equation originated. That is the primary goal of this note. This scheme was suggested by G3UUR in a letter.

Neglecting biasing details, the oscillator is shown in Fig 1 below.



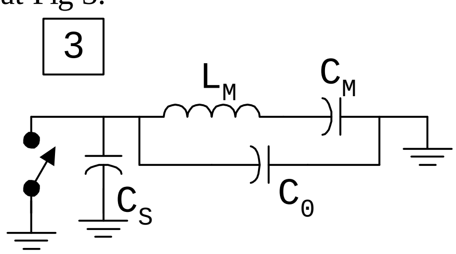
When this circuit is in oscillation, the load imposed by the transistor is fairly small and can be ignored. The capacitor at the collector is just a bypass of large value and does not impact the oscillator frequency. The parallel capacitors shown as Cp are large in value compared with any of the capacitors in the crystal circuit. Typical values might be 470 to 1000 pF. Cs is a series capacitor that can be short circuited with a switch. A typical value might be 33 pF. Throwing the switch might produce a 2 kHz shift with a 10 MHz crystal. Cs includes the parallel capacitance of the open switch, which can be several pF.

The circuit used to calculate resonances from known motional parameters is presented in Fig 2.



This circuit includes the motional elements of the crystal.

Evoking the assumption that the C_p is very large and does not alter resonance, we arrive at Fig 3.



For the time being, assume that we can neglect the parallel capacitance, C_0. This leaves us with the simple circuit of Fig 4.

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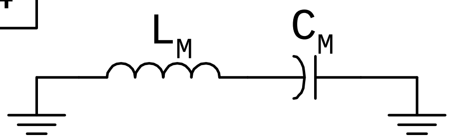

$$\omega_4^{\,2} = \frac{1}{L_m \cdot C_m}$$

This figure now includes an equation for the resonant angular frequency, ω_4, where the subscript 4 just represents the figure number.   This represents the case where the switch of Fig 1 is closed, removing C_s from the circuit.

Figure 5 shows the circuit with the switch open.

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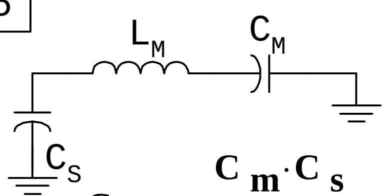

$$C_e = \frac{C_m \cdot C_s}{(C_s + C_m)}$$

$$\omega_5^{\,2} = \frac{1}{L_m \cdot \frac{C_m \cdot C_s}{(C_s + C_m)}}$$

Two capacitors are in series in this circuit, so they have an effective capacitance Ce resulting in a slightly higher angular frequency ω_5.

C_s, the switched capacitance, is known from measurement.   Similarly, we measure the frequencies of oscillation with a counter.   This leave us with two equations for ω_4 and ω_5 in the two unknowns L_m and C_m.

If we subtract one equation from the other we obtain

$$\omega_5^{\,2} - \omega_4^{\,2} = \frac{1}{L_m \cdot \frac{C_m \cdot C_s}{(C_s + C_m)}} - \frac{1}{L_m \cdot C_m}$$

This simplifies to become

$$\omega_5^{\,2} - \omega_4^{\,2} = \frac{1}{L_m \cdot C_s}$$

or

$$\frac{1}{L_m} = C_s \cdot \left(\omega_5^{\,2} - \omega_4^{\,2}\right)$$

But the motional L is related to the angular frequency with the switch closed shown in Fig 4 above,

$$\frac{1}{L_m} = C_m \cdot \omega_4^{\,2}$$

This gives us two equations for $1/L_m$. Eliminating L_m between them yields

$$C_m = C_s \cdot \frac{\left(\omega_5^{\,2} - \omega_4^{\,2}\right)}{\omega_4^{\,2}}$$

or

$$C_m = C_s \cdot \left(\frac{\omega_5^{\,2}}{\omega_4^{\,2}} - 1\right)$$

We now redefine the upper frequency as the sum of the lower and a frequency shift,

$$f_5 = f_4 + \delta f$$

Using this form, and the usual definitions

$$\omega_4 = 2 \cdot \pi \cdot f_4 \qquad \omega_5 = 2 \cdot \pi \cdot \left(f_4 + \delta f\right)$$

If we substitute these into the expression for C_m, we obtain

$$C_m = \left[C_s \cdot \delta f \cdot \frac{\left(2 \cdot f_4 + \delta f\right)}{f_4^{\,2}} \right] = \frac{C_s \cdot \delta f}{f_4} \cdot \left(2 + \frac{\delta f}{f_4}\right)$$

There are two terms on the right side. One is just the integer 2 while the other is a ratio of two frequencies. But recall that a typical value for δf is 2 kHz for f_4=10 MHz. The ratio is much less than 2, so we ignore it, which leaves

$$C_m = 2 \cdot C_s \cdot \frac{\delta f}{f}$$

We have dropped the "4" subscript with the understanding that the crystal series resonance is the defining oscillation frequency. The two oscillator frequencies are so close to each other that it makes no difference which is used.

We chose to ignore C_0 in this derivation above. The actual circuit is that of Fig 6, below.

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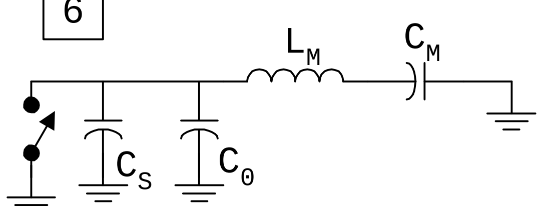

This is just a redrawing of Fig 3. But C_0 is merely in parallel with C_s. Hence, a better form for the equation would be

$$C_m = 2 \cdot \left(C_s + C_0\right) \cdot \frac{\delta f}{f}$$

Having motional capacitance, the motional inductance is easily calculated from the series resonant frequency. Parallel capacitance, C0, is easily measured with a lower frequency bridge. I usually use an AADE LC Meter. Alternatively, one can obtain an approximate value with $C_0=220\cdot C_M$. This relationship follows from the physics of the AT-Cut quartz crystal, but does not include capacitance of any package that might hold the crystal. Perhaps a better guess would be $C_0=220\cdot C_M+1$ pF.

As mentioned earlier, the simple oscillator scheme for determining motional parameters was suggested to me in a 1982 or 83 letter from Dr. David Gordon-Smith, G3UUR. Jack Smith, K8ZOA, pointed out that I really needed to include parallel capacitance with the switched capacitance in the formula for motional inductance. Thanks to both of these experimenters.