

Designing High Order LC Bandpass Filters from EMRFD

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The information presented in Experimental Methods in Radio Frequency Design concerning LC Filters concentrates on the double and triple tuned circuits. These filter forms are often adequate for many applications. However, there is sometimes a need for a LC filter of order beyond 2 or 3. In this note we will present the simple equations needed for designing a double tuned circuit. We will then show how the same set of equations, and any program that might be written around it, can be used to design a higher order filter. The method is presented through an illustration with a 4th order LC filter for the amateur 80 meter band. The equation set and software distributed with EMRFD will be used in this exercise.

Often there is an interest in building a preselector filter. This is a circuit that can be tuned over a fairly wide range with approximately a constant bandwidth.

We will show how software attached to EMRFD is used to evaluate such a preselector.

We begin with the equation set presented in a sidebar on page 3.14 of EMRFD.

In this example we have put the equations into MathCad. These equations are easily read and used to form your own programs for a programmable hand calculator or a computer. Numerous programming languages can be used. We recommend that you do your own program in a format that is familiar to you. The MathCad work sheet is shown below.

Double Tuned Circuit Design, Sidebar from EMRFD, p 3.14.

Input Variables:

$$f_m := 7.1 \quad L_u := 1.5 \quad b_m := 0.2 \quad k := 0.707 \quad q := 1.414$$

$$Q_u := 200 \quad R_0 := 50$$

Now, the design Equations:

$$f := f_m \cdot 10^6 \quad \omega := 2 \cdot \pi \cdot f$$

$$L := \frac{L_u}{10^6} \quad b := b_m \cdot 10^6$$

$$C_0 := \frac{1}{(\omega^2 \cdot L)} \quad C_{12} := C_0 \cdot \frac{k \cdot b}{f} \quad Q_e := \frac{q \cdot f \cdot Q_u}{b \cdot Q_u - q \cdot f}$$

$$C_e := \frac{1}{\omega} \cdot \frac{1}{\sqrt{R_0 \cdot Q_e \cdot \omega \cdot L - R_0^2}} \quad C_t := C_0 - C_e - C_{12}$$

Going through these equations will yield the filter. Now we need a few equations that will convert the Farad results back to picofarads.

$$C_{12p} := C_{12} \cdot 10^{12} \quad C_{tp} := C_t \cdot 10^{12} \quad C_{ep} := C_e \cdot 10^{12} \quad C_{0p} := C_0 \cdot 10^{12}$$

And now, our outputs in pF:

$$C_{12p} = 6.671 \quad C_{tp} = 280.714$$

$$C_{ep} = 47.605 \quad C_{0p} = 334.99$$

We will not discuss the equations in any detail, for they are presented in the book. However, we note that in this double tuned circuit example we have included the k and q values for a Butterworth filter. The k parameter is a normalized coupling coefficient while the q is a normalized Q for the end sections. Denormalization occurs for a desired bandwidth and center frequency. Different k and q will yield new filter shapes. The parameters at the top of the work sheet are *input* parameters including center frequency and bandwidth as well as a description of the inductors that will be used to realize

the filter. There is a wide variety of L values that will work in such designs.

This is NOT a case where L is predicted by the design process. Rather, it is a user defined parameter. In the example we consider a 7.1 MHz filter with a bandwidth of 0.2 MHz. We have picked inductors of 1.5 μH (1500 nH) with an unloaded Q of 200. This is a value easily realized at 7 MHz with a toroid core.

This is a simplified version of the equation set used in DTC.exe and TTC.exe, the Windows based programs distributed with EMRFD. These programs design double or triple tuned Butterworth filters. The reader can readily verify that the two are equivalent.

Assume now that we wish to design a bandpass filter with a center frequency of 3.7 MHz and a bandwidth of 0.1 MHz. Moreover, we wish to use four resonators and realize a shape of a 0.1 dB ripple Chebyshev filter. We could look up the appropriate k and q values from a suitable table of filter parameters.

Alternatively, the data for Chebyshev and Butterworth filters is built into one of the programs. To generate this data, go to XLAD.EXE. Merely accept the default data and click on steps 1 and 2 of the program. Then click on step 2A, which will take you to a new window. When there, enter $N=4$ and $\text{ripple}=0.1$ dB. The program then generates $q=1.35$, $k_{12}=k_{34}=0.685$, and $k_{23}=0.542$.

Write these parameters down for later use and then exit XLAD.

We now return to our equation set and enter new *input* parameters. We use a termination of 50 Ohms, allowing the filter to be easily aligned and evaluated with available instrumentation. An inductor was needed. We looked in the junk box and found a good sample, a T68-2 core with 51 turns of #26 enameled wire. We measured the inductance at 15.16 μH with a Q of 231. See the Q measurement discussion in Chapter 7 of EMRFD. We repeated the L measurement with an AADE L/C meter to get 14.95 μH . The new work sheet is shown below with the required data now inserted.

Double Tuned Circuit Design, Sidebar from EMRFD, p 3.14.

Input Variables:

$$f_m := 3.7 \quad L_u := 15 \quad b_m := 0.1 \quad k := 0.685 \quad q := 1.35$$

$$Q_u := 230 \quad R_0 := 50$$

Now, the design Equations:

$$f := f_m \cdot 10^6 \quad \omega := 2 \cdot \pi \cdot f$$

$$L := \frac{L_u}{10^6} \quad b := b_m \cdot 10^6$$

$$C_0 := \frac{1}{(\omega^2 \cdot L)} \quad C_{12} := C_0 \cdot \frac{k \cdot b}{f} \quad Q_e := \frac{q \cdot f \cdot Q_u}{b \cdot Q_u - q \cdot f}$$

$$C_e := \frac{1}{\omega} \cdot \frac{1}{\sqrt{R_0 \cdot Q_e \cdot \omega \cdot L - R_0^2}} \quad C_t := C_0 - C_e - C_{12}$$

Going through these equations will yield the filter. Now we need a few equations that will convert the Farad results back to picofarads.

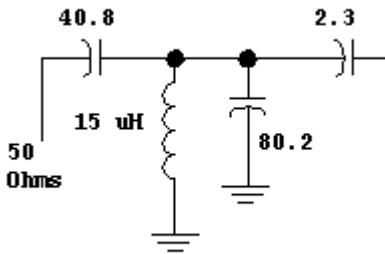
$$C_{12p} := C_{12} \cdot 10^{12} \quad C_{tp} := C_t \cdot 10^{12} \quad C_{ep} := C_e \cdot 10^{12} \quad C_{0p} := C_0 \cdot 10^{12}$$

And now, our outputs in pF:

$$C_{12p} = 2.284 \quad C_{tp} = 80.241$$

$$C_{ep} = 40.827 \quad C_{0p} = 123.352$$

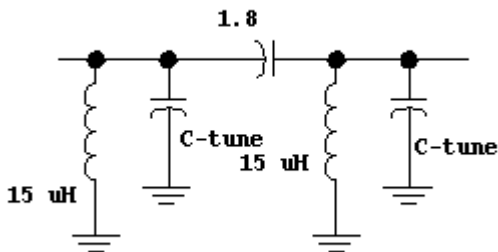
Note that we have used $k=k_{12}=0.685$ and $q=1.35$, parameters related to the end of the filter. So, the work sheet now generates the end parameters and the



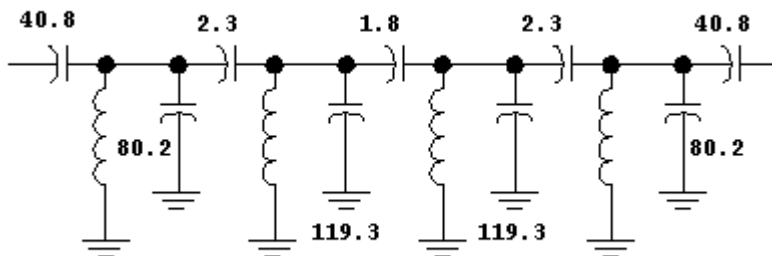
related partial circuit:

Both ends are the same, for the q and k parameters are equal.

Now having the ends designed, we change the k in the work sheet to obtain the coupling capacitor between inner sections. The partial circuit is then

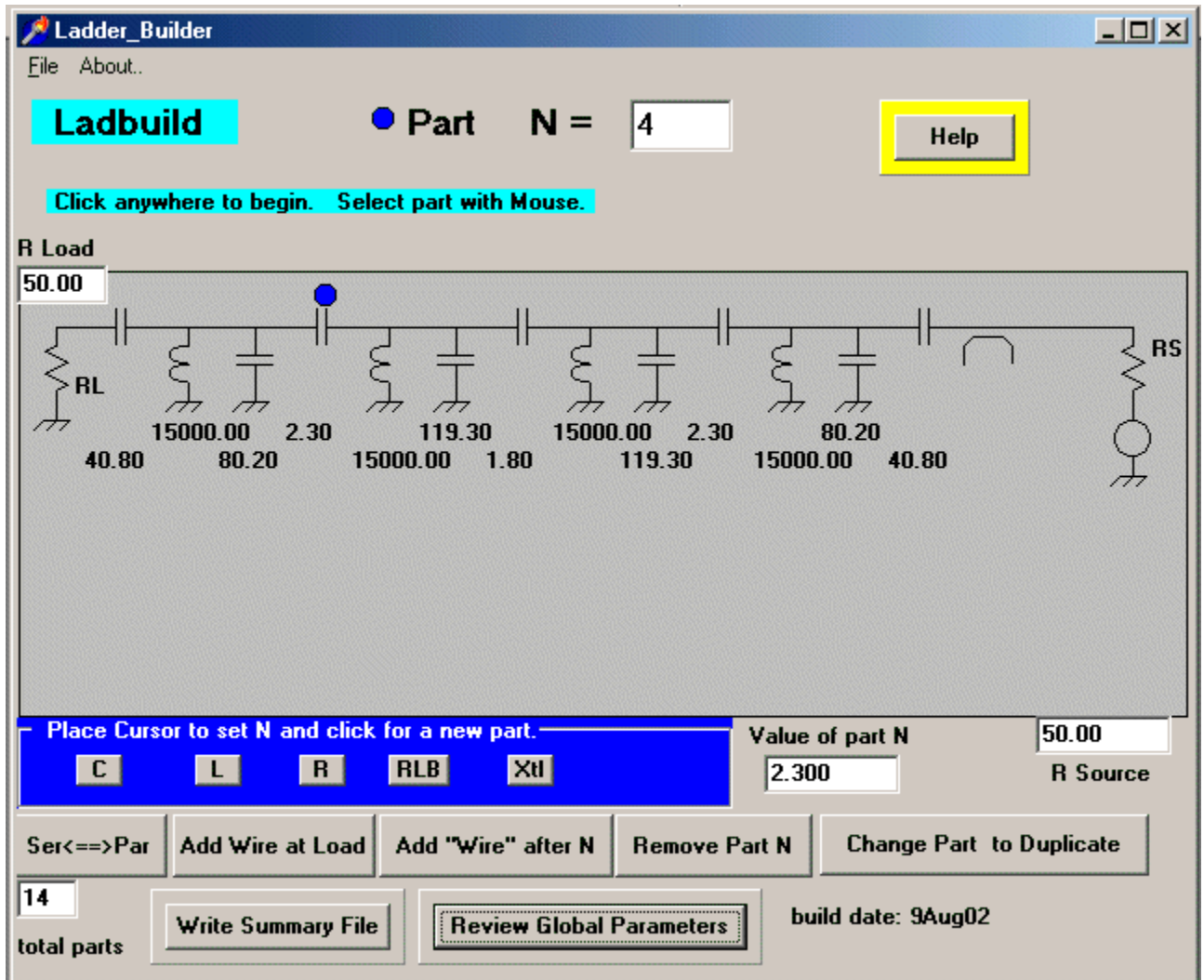


The tuning capacitor is not calculated, but the form is simple. The tuning capacitance is merely the nodal capacitance (generated by the work sheet) less the capacitors loading the resonator to be tuned. Using this, the inner tuning capacitors are 119.3 pF. The overall filter is then



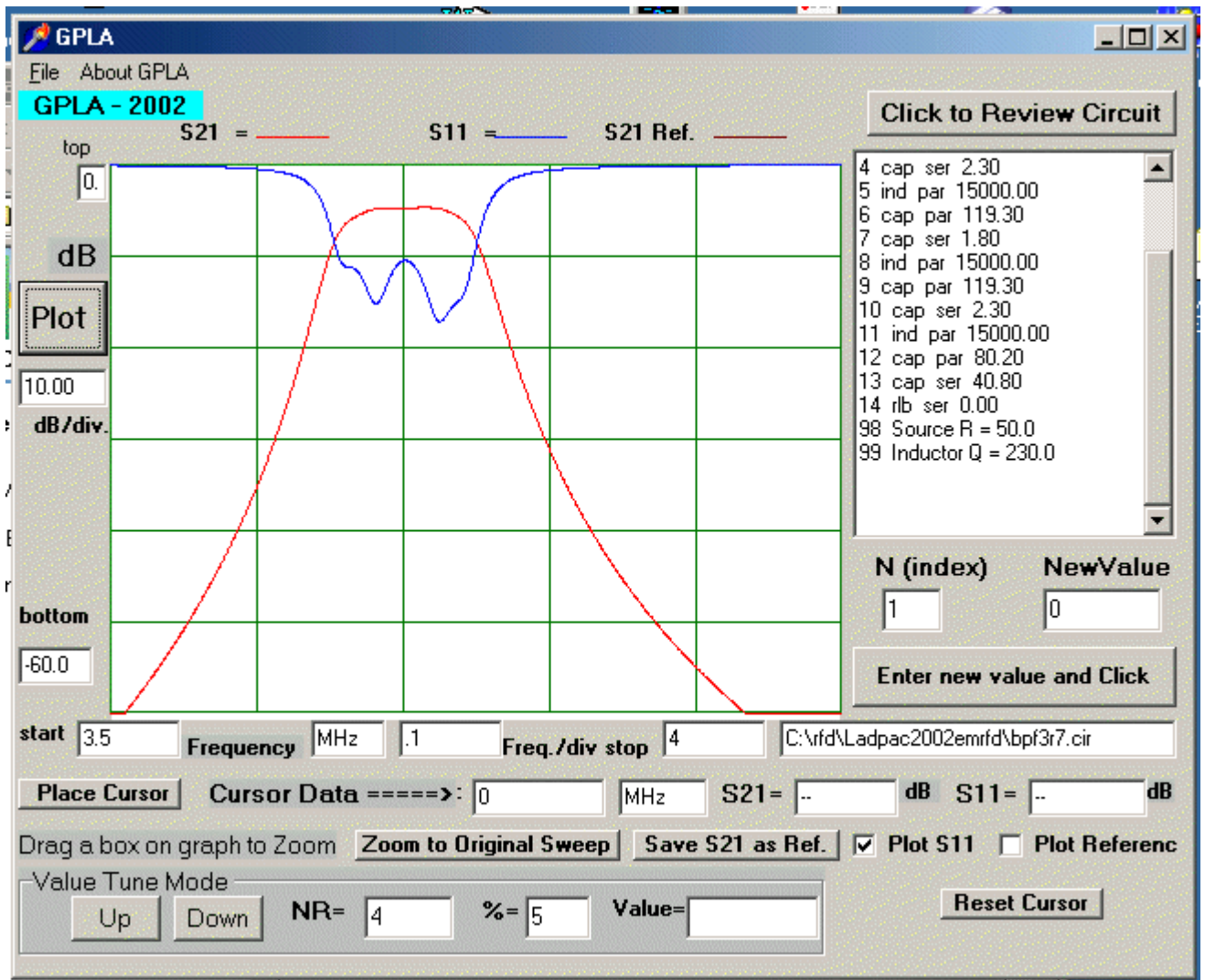
We now wish to use the General Purpose Ladder Analysis to evaluate the filter we have just designed. We must first get the circuit into the program, which is done with the circuit editor, Ladbuild.exe. This program is called and we then click on **file** and **new**. This gives us a blank circuit, containing only terminations and a return loss bridge. We place the cursor on position number

1 and enter a series capacitor. We set the value to 40.8 pF in the value window and then place the cursor in the second position. Moving the cursor fixes the value just entered. This is repeated through the filter, producing a final circuit.

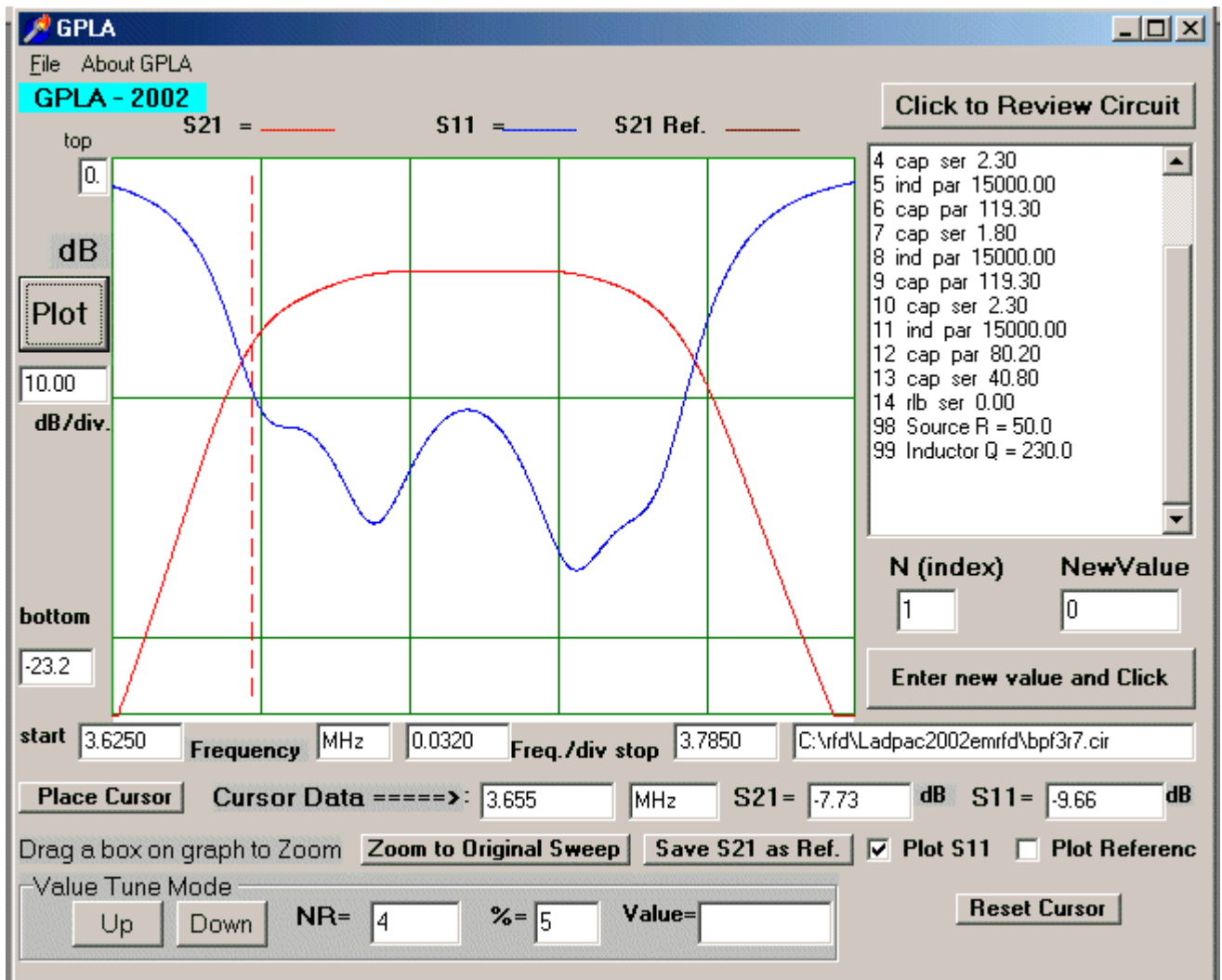


We must also use the "Global Parameters" window to set the Q at 230. We also set up a sweep from 3.5 to 4 MHz with lines at 0.1 MHz intervals. The file is then saved as bpf3r7.cir.

With the file saved, we minimize the schematic editor window and call GPLA.exe. We then open the file and display the result, shown below.

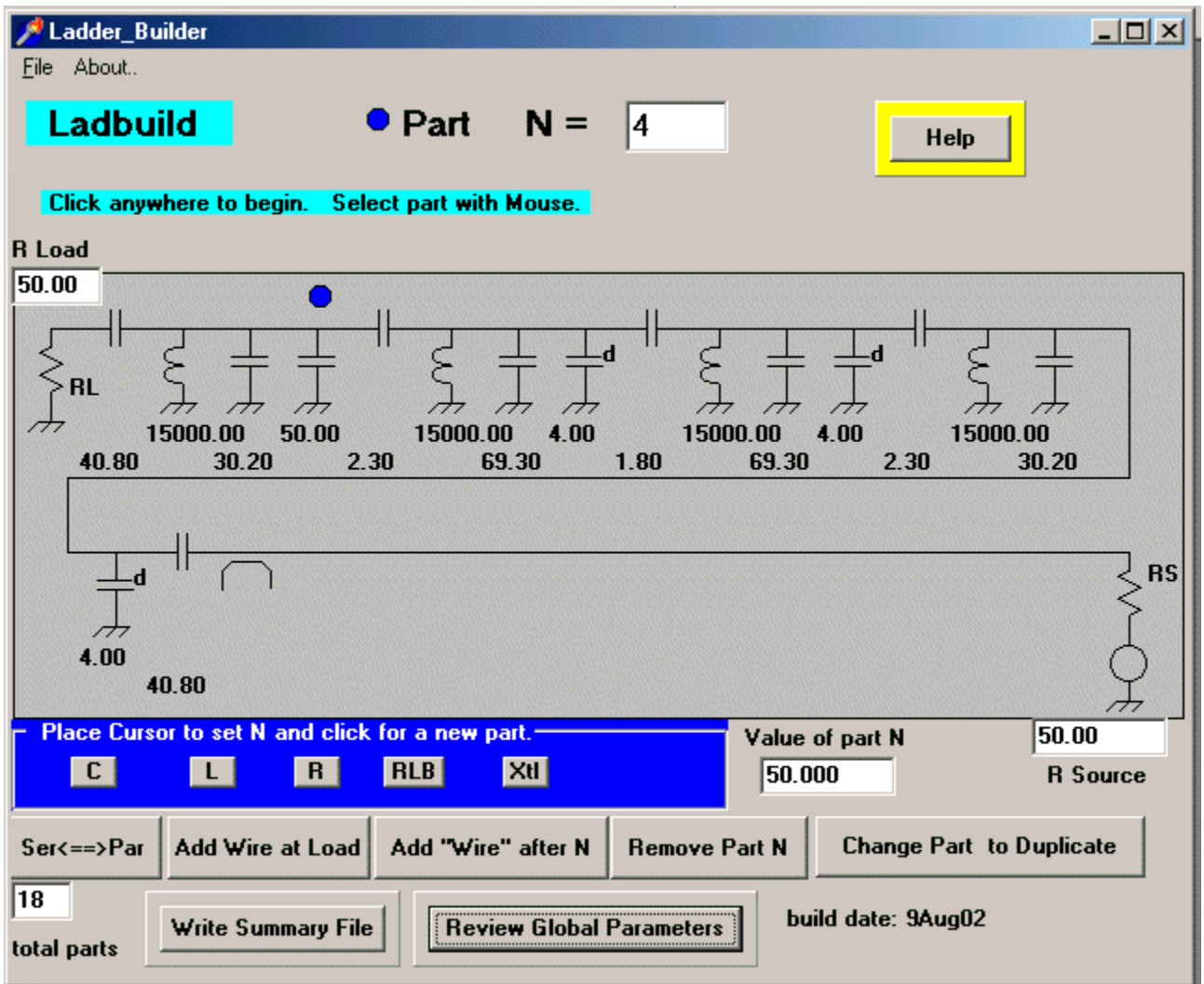


We can now zoom in and look at the peak in detail. The cursor function can be used to evaluate the insertion loss at 4.75 dB and to establish the 3 dB bandwidth as about 94 kHz. We can also change the unloaded inductor Q to a very high value (e.g., 5000) and see a flat Chebyshev like response.

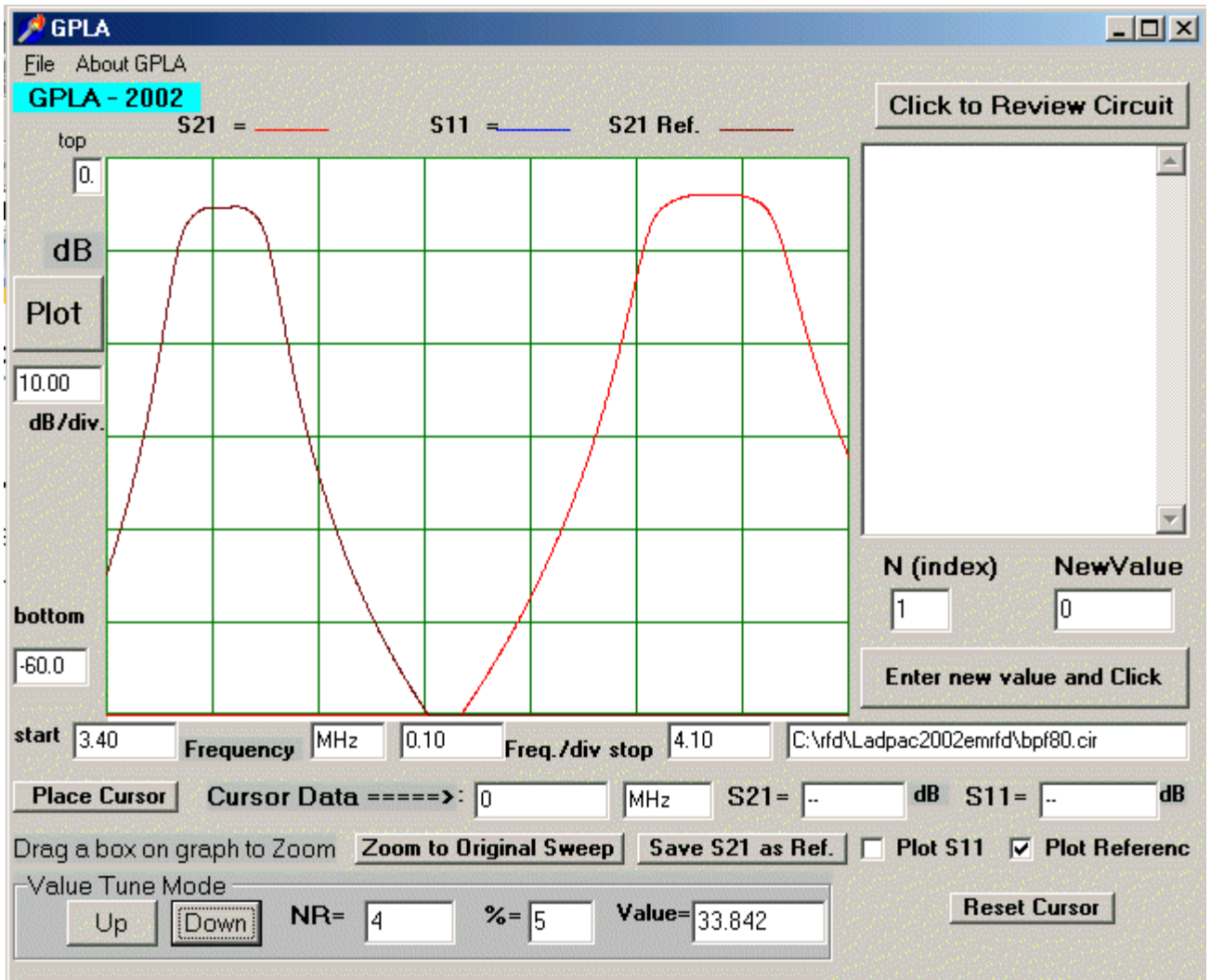


This filter would be a practical one. We could build it with 39 pF end capacitors and coupling caps of 2.2, 1.8, and 2.2 pF. The tuning capacitors are combinations of fixed caps with trimmers.

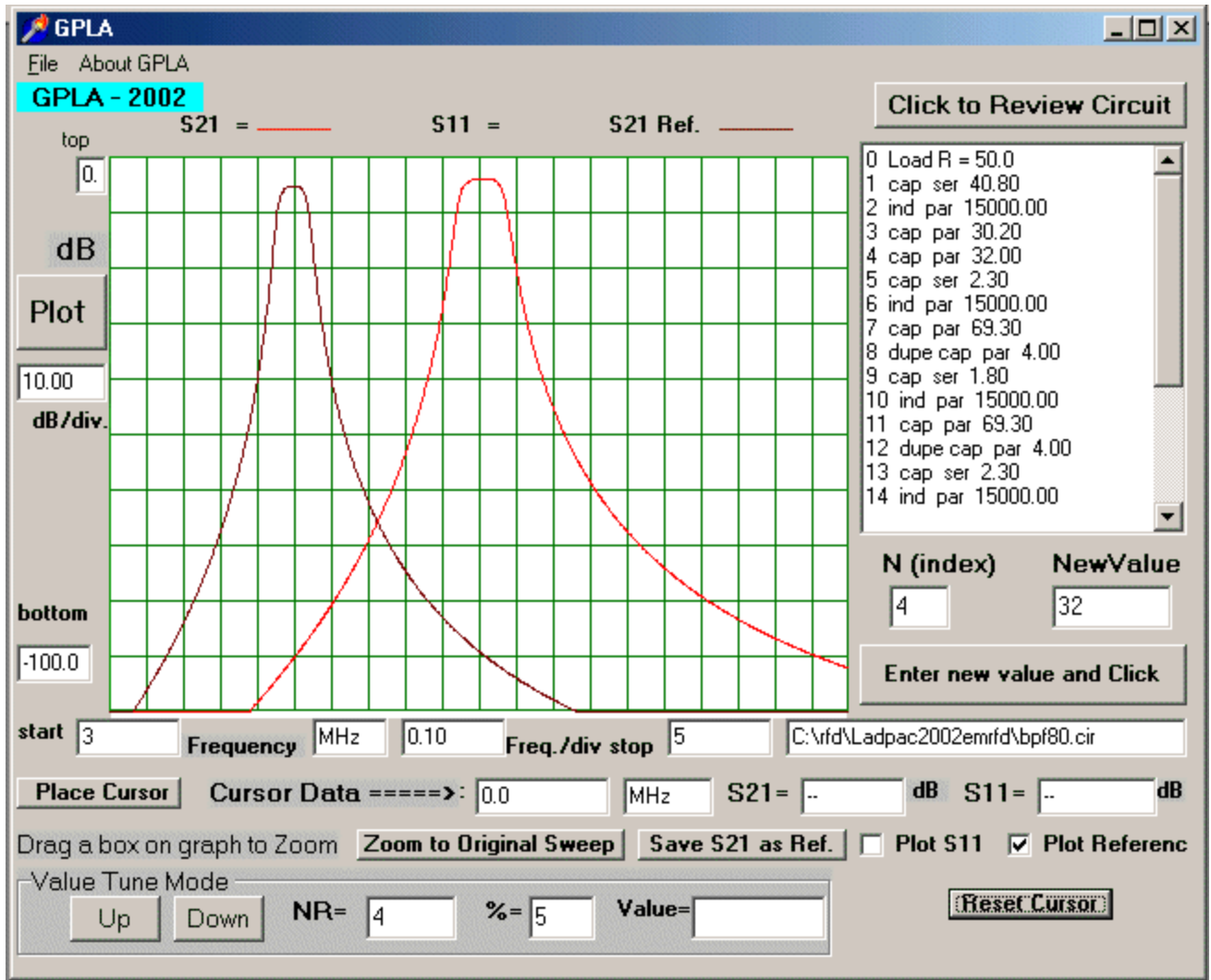
Next, we ask about tuning this circuit over the entire 3.5 to 4 MHz region with a multi-section variable capacitor. Four section variables are rare, but show up on the surplus market from time to time. To simulate this situation, we begin by reducing the value of all four tuning capacitors by 50 pF. We then use the "add wire after N" function in Ladbuild.exe to insert blank wires in parallel with each of the former tuning capacitors. The first capacitor, inserted in position 4, has a value of 50 pF. Parallel capacitors are inserted in the three remaining slots, but are then converted to "duplicate parts." A "value" of 4 is used, indicating that they are duplicates of part number 4. We have now created a 4 section ganged variable capacitor. See the circuit below.



The circuit is saved and loaded into GPLA. An initial sweep shows exactly the same response as seen earlier. However, we can now change the value of part number 4 and watch the response move across the band. Clicking the "save S21 as ref" creates a reference or "remembered" sweep. We can then tune the filter to a new center and plot both responses. This is shown below for C values of about 34 and 65 pF.



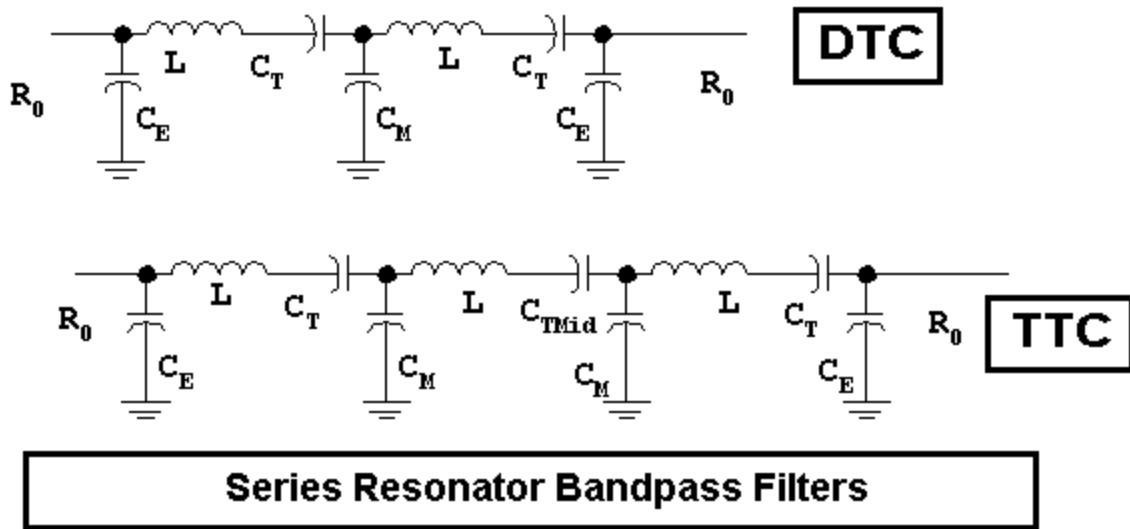
Another variation shows stopband attenuation characteristics for two sweeps.



Note that there is a change in insertion loss as the filter is tuned. This results from using capacitors for both tuning and for coupling between resonators and to the outside world. If links are used at the input and output and if inductors are used as coupling elements, but retaining variable C for tuning, a substantially flat insertion loss will result.

Filters with Series Resonators:

Another form of bandpass filter of interest is one using series resonators. This topology does not lend itself to preselector applications, but is excellent for fixed tuned filters. Double and triple tuned filters are shown here:



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A MathCad worksheet set up for the double tuned circuit is presented below. Actually, this worksheet will design either the double or the triple tuned variation. The k and q data for Butterworth shape are presented. The equations for calculating the middle tuning capacitor of a triple tuned circuit is included. This program, or one much like it, was used to generate the filters used with the receiver part of the Lichen transceiver of EMRFD, Chapter 6.

Double and Triple Tuned Circuits with Series Resonators. IRFD p92.

$$f := 3.75$$

$$L_n := 15000$$

$$Q_u := 50$$

$$B := 0.6$$

$$\omega := 2 \cdot \pi \cdot f \cdot 10^6$$

$$L := L_n \cdot 10^{-9}$$

$$Q_f := \frac{f}{B}$$

$$C_0 := \frac{1}{(\omega^2 \cdot L)}$$

$$q = 1.414 \text{ for DTC, } 1.0 \text{ for TTC.}$$

$$q := 1.0$$

$$q_0 := \frac{Q_u}{Q_f}$$

$$k := 0.7071$$

<==== holds for both DTC and TTC.

$$Q_e := \frac{-1}{\left(\frac{1}{Q_u} - \frac{1}{q \cdot Q_f} \right)}$$

$$R_0 := 50$$

$$K := \frac{k}{Q_f} \quad C_m := \frac{C_0}{K}$$

$$R_s := \omega \cdot \frac{L}{Q_e}$$

$$C_e := \sqrt{\frac{R_0 - R_s}{R_s \cdot \omega^2 \cdot R_0^2}}$$

$$c := \frac{C_e^2 \cdot \omega^2 \cdot R_0^2 + 1}{C_e \cdot \omega^2 \cdot R_0^2}$$

$$C_t := \left(\frac{1}{C_0} - \frac{1}{c} - \frac{1}{C_m} \right)^{-1}$$

$$S_{21} := -20 \cdot \log \left(\frac{q_0}{q_0 - q} \right) \quad \text{Approx}$$

Remember that decreasing the mutual capacitor values produces higher coupling, consistent with wider bandwidth.

The figures above illustrate a triple tuned circuit with a bandwidth of 600 kHz. When we tried to use 700 kHz, the results presented were some capacitors with values that were complex, indicating that we tried to take a square root of a negative value.

The MathCad program was modified to use Chebyshev k and q values, shown in the following table:

Chebyshev k and q, n=3

Ripple	k	q
0 dB	0.707	1.0 (Butterworth)
0.01	0.682	1.18
0.1	0.662	1.43
0.3	0.651	1.69
1.0	0.644	2.22

The modified MathCad Worksheet is shown below where we consider the same filter at 3.75 MHz with low Q inductors:

Triple Tuned Circuits with Series Resonators. Chebyshev, 1 db PBR

$$f := 3.75 \quad L_n := 15000 \quad R_0 := 50 \quad Q_u := 50 \quad B := 1.34$$

$$\omega := 2 \cdot \pi \cdot f \cdot 10^6 \quad L := L_n \cdot 10^{-9} \quad Q_f := \frac{f}{B} \quad q_0 := \frac{Q_u}{Q_f}$$

$$q := 2.22 \quad k := 0.644 \quad Q_e := \frac{-1}{\left(\frac{1}{Q_u} - \frac{1}{q \cdot Q_f} \right)} \quad C_0 := \frac{1}{(\omega^2 \cdot L)}$$

$$K := \frac{k}{Q_f} \quad C_m := \frac{C_0}{K} \quad R_s := \omega \cdot \frac{L}{Q_e} \quad C_e := \sqrt{\frac{R_0 - R_s}{R_s \cdot \omega^2 \cdot R_0^2}}$$

$$c := \frac{C_e^2 \cdot \omega^2 \cdot R_0^2 + 1}{C_e \cdot \omega^2 \cdot R_0^2} \quad C_t := \left(\frac{1}{C_0} - \frac{1}{c} - \frac{1}{C_m} \right)^{-1}$$

$$\text{Mutual C} : C_m \cdot 10^{12} = 521.828 \quad \text{pF}$$

$$\text{End C} = C_e \cdot 10^{12} = 51.062 \quad \text{pF}$$

$$\text{End Tuning C} = C_t \cdot 10^{12} = 157.716 \quad \text{pF}$$

$$C_{\text{mid}} := \left(\frac{1}{C_0} - \frac{2}{C_m} \right)^{-1} \quad \text{Middle tuning cap} = C_{\text{mid}} \cdot 10^{12} = 222.48 \quad \text{pF.}$$

This example uses the 1 dB passband ripple k and q parameters. It is now possible to extend the bandwidth to 1.34 MHz without seeing the complex capacitor values. The general result is that the higher the passband ripple, the wider the bandwidth can become.

The example above used a design Q of 50. A SPICE (Linear Technology variation) run was performed to illustrate the effects. For this example, Q was increased to 500, producing a filter with very low loss. The SPICE file is shown here:

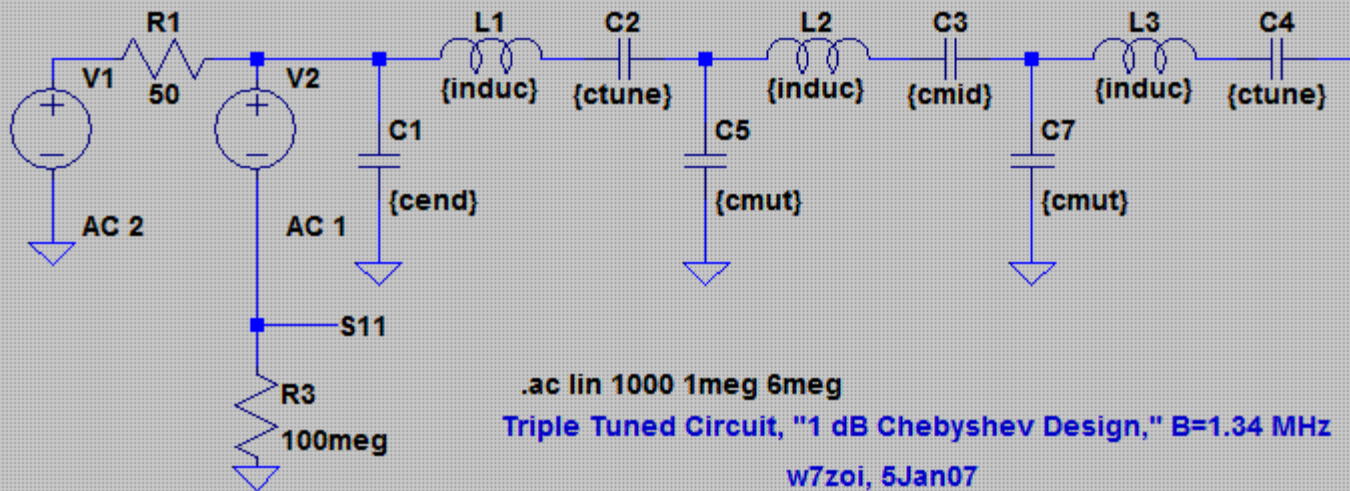

```

.param induc 15u
.param Q 500
.param f0 3.75
.param rs=6.28*f0*1000000*induc/Q

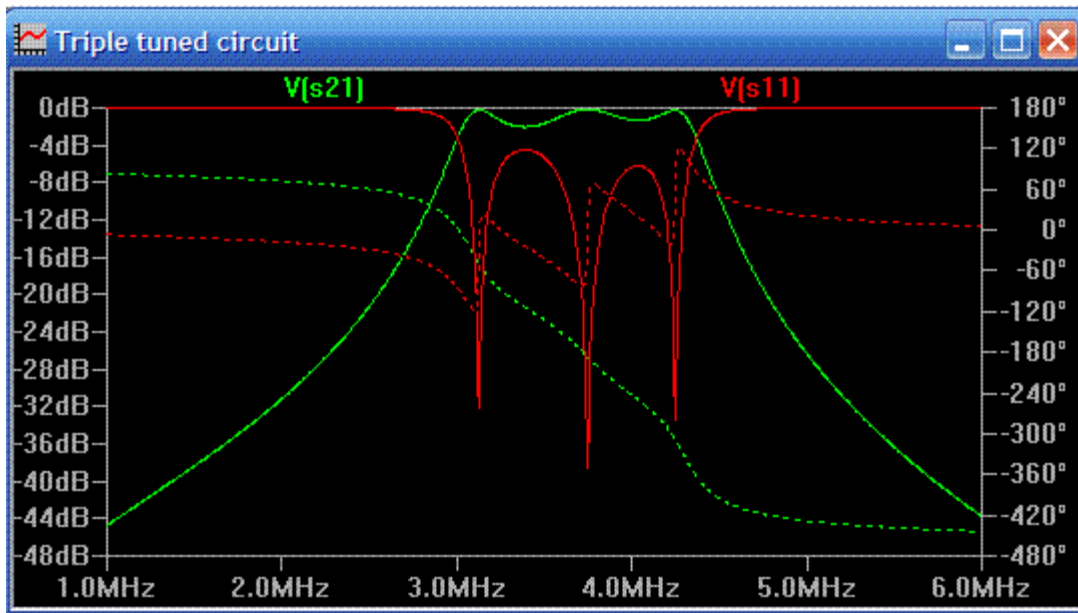
.param cend 51p
.param cmut 522p
.param ctune 158p

.param cmid 222.5p

```



Essentially all of the filter design results are set up as "param" statements, which lets us make changes and see the resulting behavior. The result with the high Q of 500 is :



We elected to use SPICE for the analysis merely to illustrate the utility of this familiar tool. (Thanks to Linear Technology, Inc., for making this program available.)

One of the most interesting details to emerge from this analysis was that the ripple in this filter is much larger than the design value of 1 dB. This is the result of the narrowband transformations (low pass to bandpass) that are used in this design. The approximations that applied near the center of 3.75 MHz no longer work in this wideband case. Other synthesis methods may well be better if accurate filter response is important. Often, such details are not that important. That is, all we often are concerned about is allowing some ripple to exist in the passband while having skirts that are as steep as reasonable with a stopband attenuation that remains high.

Although we won't go through the details here, it should be evident that these worksheets can be used to design even higher order filters. The filter with series tuned resonators tends to have a much cleaner stopband, especially in the lower parts of the HF (3 to 30 MHz) spectrum.

Readers who own a copy of MathCad and would like a copy of the worksheets should send me email and I'll email them copies of the files. I'm now using Version 2001i.

